Angelo Guerraggio
Giovanni Paoloni

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Translated by Kim Williams

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Angelo Guerraggio
Department of Decision Sciences
University of Bocconi
Milano, Italy

Giovanni Paoloni
"La Sapienza"
University of Rome
Rome, Italy

Translator
Kim Williams
Kim Williams Books
Turin, Italy

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Introduction

The Italian mathematics community experienced what is probably its most brilliant period during the decades at the turn of the twentieth century. When the different states of the peninsula were united in 1861, annexing them to what had been the Kingdom of Sardinia and would go on to become the Kingdom of Italy, the field of mathematics already boasted several illustrious scholars. However, apart from these individuals, it struggled to be seen as a large and recognised entity. In the course of thirty to forty years, the situation changed radically, and Italy's being assigned as the third venue of the International Congress of Mathematicians (held in Rome in 1908, following the 'rehearsal' in Zurich in 1897, and the congresses in Paris and Heidelberg in 1900 and 1904) sanctioned its new role within the international community. In his report of the Rome congress for the French newspaper *Le Temps*, Henri Poincarè wrote: 'For thirty years now, the Italian mathematical movement has worked very intensely, in Rome as well as in universities in other provinces: I could cite a large number of names who occupy an honourable place in the history of sciences, but seeing them gathered together in this Congress makes it much easier to see how active Italian scientific life has been'.

The rapid, and in some ways surprising, developments in Italian mathematics were not accomplished solely on terrain that was properly scientific and through relations with the scholars most acclaimed on the international level, but were also accompanied by the increasingly large influence that mathematicians exerted on Italian society. They dealt with problems of education at all levels of schools, not only universities (it is easy to imagine the criticism that the newly created government of Italy had to face in creating a national system of education); they were active within other scientific communities; they engaged in dialogue and an exchange of ideas – not always harmonious – with the world of philosophy; they intervened in national politics; they placed their experience at the service of accelerating the modernisation and industrialisation of the country. In short, this was what has been referred to as the *spring* or *golden age* of Italian mathematics.

Taking part in this were outstanding researchers such as Guido Castelnuovo, Federigo Enriques, Francesco Severi, Luigi Bianchi, Giuseppe Peano, Salvatore Pincherle, Giuseppe Vitali, Leonida Tonelli, Guido Fubini, Gregorio Ricci-Curbastro,

Tulli Levi-Civita and others, some already established, and some who would realise their full potential during the period between the two world wars. Even Poincarè, in his *Le Temps* report, was hesitant to name names, 'because I fear, in fact I am sure, that I will forget someone important'.

Among the remarkable figures just named, that of Vito Volterra stands out. It is no overstatement to call him one of the greatest Italian mathematicians of the day, certainly the most influential, and noteworthy for both his international relations and the duties he performed out in academic and political institutions. For this reason the story of his life makes it possible to come into contact with some of the most significant research in analysis and mathematical physics of the final decades of the nineteenth century and first decades of the twentieth, and to span a long period – up to the years just before the tragedy of World War II – of events in the international mathematics community, the fates of the scientific institutions in various countries as they grappled with the problems, opportunities and expectations that emerged from the previous world war, and the travails of Italian political life during the crucial period of the passage from liberal governments to the Fascist regime.

Of particular significance in Volterra's mathematical formation is the moment when the young student of the Scuola Normale in Pisa decided to focus on the physics-mathematics teaching of Enrico Betti rather than that of Ulisse Dini, the greatest follower in Italy of the rigorism of German analysts, who had provided the orientation for his studies during his first two years of university. This would mean that the Notes written by Volterra, at the time still a student at the Scuola Normale, on pointwise discontinuous functions and the relationships between derivatives and Riemann integrals (Notes which were crucial for the history of the theory of integration) would have no sequel. Instead, there would shortly appear the Memoirs on functional analysis, and the articles on integral and integro-differential equations, all accompanied by a concern for showing how the most abstract mathematical formalism, thought of as free to travel unexplored paths, was in reality always somehow oriented by the study of nature and by the urgency or curiosity to get to the bottom of some physical problem.

This balance between basic, curiosity-driven research and applications (not only to physics), surely a legacy of his apprenticeship with Betti, constitutes one of Volterra's most important lessons in mathematics, even at a distance of more than a hundred years. We find deep traces of it in his lectures, both official and for a general public; in his interest in the then young field of mathematical economics; in his "Leçons sur la théorie mathématique de la lutte pour la vie", which along with the so-called Lotka-Volterra model, elaborated by a mature Volterra, inaugurated a new phase in the study of population dynamics; in his correspondence with colleagues, where he calmly but firmly reiterates (for example, in some of the letters written to Maurice Fréchet) the correctness of his choice not to pursue generalisations that seemed to him to be ends in themselves. This is the conviction of one who does not limit himself to proving theorems, but constructs theories, and wants these theories to have meaning, not without, naturally, asking himself what meaning a mathematical theory can have.

Then there is Volterra involved in society, not entering directly into the political arena, but using the authoritativeness acquired in the field of mathematics to remind Italian society what science is and what it can offer for the progress of the nation. In fact, his faith in science becomes a social commitment. This is Volterra the Senator of the Kingdom; it is Volterra the founder, at the beginning of the twentieth century, of the Italian Association for the Advancement of Science, with which he calls researchers and scholars to a greater awareness of the role that they can play and which politics must sooner or later recognise; it is Volterra the founder, immediately after the first world war, of Italy's National Research Council, fruit of the work undertaken during the war and of collaboration with other allied nations to find a way to make coordinated use in peacetime as well of the advantages offered by science and technology, and of the attempt to make science the basis of a new model for Italy's future development.

By now we are many decades away from the experience of Fascism, and yet the lesson drawn from the stand taken by Volterra seems more relevant than ever. He was a moderate, and would remain so his whole life. Rather, his anti-Fascism was based on an opposition to the tyranny with which politics – Mussolini's politics – sought to impose its laws on the world of knowledge, and on the conviction, moreover, that such tyranny was unfounded. Volterra's judgment on the political class in the form it took under the regime was harsh. His refusal to take the oath of allegiance imposed on university professors by Fascism in 1931 remains an unequivocal word of warning. He knew full well that the majority of his colleagues – only twelve would refuse to swear allegiance – would give in and obey, either out of conviction, indifference, opportunism or desperation. He also knew full well that his refusal could have no other immediate outcome than his being fired from the university, or as it turned out, forced into early retirement. And yet, he couldn't bring himself to swear. He believed that there are moments in which one can and must say no.

Chapter 1
Chronicles of Italian Life

1.1 The Family

In the spring of 1860, Ancona was living out its final months under the dominion of the Papal States, which was destined to end the following September. It was in this city in the Marches that Vito Volterra was born, on 3 May 1860.

His parents were members of the Jewish community, which at that time lived in the ancient ghetto established in the sixteenth century.[1] His father, Abramo, came from a family of local merchants. His mother, Angelica Almagià, was the niece of Saul, one of the notables of the community. After losing her father when she was 7 years old, she grew up (together with her brother Alfonso) in close contact with the four children of his uncle Saul: Roberto, Edoardo, Vito and Virginia. This network of relationships, and in particular the close, affectionate ties of Angelica with her brother Alfonso and her cousin Edoardo, would have an profound influence on the life of the future mathematician.[2] Angelica and Abramo Volterra married on 15 March 1859, after a 2-year engagement. As was rather common at the time, their first son was given the name of his maternal grandfather.[3]

The destiny of young Vito immediately, and suddenly, changed in 1862 when his father died, leaving his young widow and child without any economic resources.

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[1]The first 'ghetto' for Jews was established in Venice in the fourteenth century; from here the name given to the confined area in which the Jews were forced to live spread throughout the world. The Jewish communities of Ancona and Rome were the only ones saved in 1569, when a decree issued by Pope Pius V expelled the Jews from the Papal State. The Ancona ghetto was abolished in 1848, but in actual fact the Jews continued to be confined to the area that had always been assigned to them, on the slopes of Monte Guasco, one of the two plateaus that make up the city of Ancona.

[2]This piece of information and others concerning Volterra's family are drawn from the book by Judith Goldstein, *The Volterra Chronicles* (Providence: American Mathematical Society–London Mathematical Society, 2007).

[3]Angelica Almagià's father, Vito Almagià (1797–1843), was a teacher in one of the Jewish schools in the ghetto.

Angelica and Vito moved in with her brother, Alfonso, who lived with his mother and unhesitatingly assumed responsibility for maintaining his sister and nephew. Vito thus grew up with his uncle, mother and grandmother. In autumn 1863, his uncle was hired by a company that contracted for the railways, and moved to Terni, along with the rest of the family. Then, in January 1865, he began a new job with the Banca Nazionale and was transferred to the central headquarters in Torino. These were the months immediately following the September Convention of 1864, in the very midst of the transfer of the capital of the fledgling Italian nation from the city in Piedmont to Firenze. In July, the Banca Nazionale also transferred, and Alfonso Almagià packed his bags one more time. He had gone to Torino alone, the rest of the family having returned to Ancona, but now his mother, sister and nephew joined him once again, and it was in the Tuscan capital that Vito Volterra would spend the years of his early education.

1.2   Uncle Alfonso's Concerns

Vito's young mother, now widowed, devoted all of her attention and affection to her child. Her upbringing, in a family with a certain level of education, made her very attentive to the pre-school education of her son, and Vito responded well to her affectionate proddings (Fig. 1.1).

   At the age of 3 he learned to write, and showed an early inclination towards studies in physics and mathematics. He himself, in an autographical memoire written many years later, told how at the age of 9 his imagination was fired by the book of popular science by Jean Macé entitled *Histoire d'une bouchée de pain* (*History of a Mouthful of Bread*), dedicated to the chemistry and physiology of human and animal life, and how, at more or less the same time, he had discovered on his own that the oscillations produced by a twisted string are isochronic, like those of a pendulum.[4] In short, young Vito began to be drawn to the observation of natural phenomenon and to his first exercises in deductive reasoning. Let there be no mistake: we are speaking about a boy just 10 years old, but all the same these first intellectual experiences – by a 10-year old boy – take on a certain significance in light of later developments. In 1873, Vito read Jules Verne's *De la Terre à la Lune* (*From the Earth to the Moon*), and decided for fun to look into what would become a classic problem of space travel in the twentieth century, and to calculate,

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[4]See 'Matériaux pour une biographie du mathématicien Vito Volterra', *Archeion* 23 (1941): pp. 325–359. This autobiographical recollection was written anonymously; the author may have been either Joseph Pérès or Elena Freda. The French mathematician Pérès, who we will learn more about in chapter VIII in particular, enjoyed a long, affectionate friendship with Volterra and his wife Virginia. Freda (1890–1978), who earned a degree in mathematics under the advisement of Guido Castelnuovo, was the author of a 1937 book (in French, with a preface by Volterra) on hyperbolic partial differential equations.
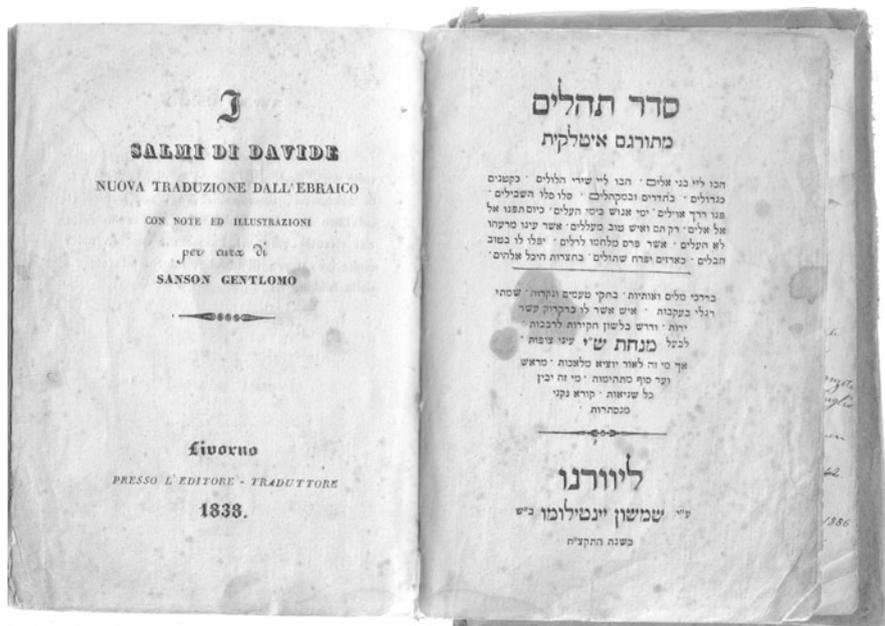
Fig. 1.1 Note recording the birth of Vito Volterra (*above right*) on the flyleaf of the Salmi di Davide (Psalms of David), translated with notes and illustrations by Sanson Gentilomo, Livorno, 1838; immediately following is a note recording the death of his father, Abramo Volterra

again for fun, the trajectory of the rocket which in the book is launched from the earth. The technique he used would lead him unawares to apply the ideas that form the basis of infinitesimal analysis. It was precisely to this adolescent experience that Volterra would refer in some passages of a lecture given in Paris in 1912 regarding the evolution of the fundamental ideas of infinitesimal calculus.[5]

Beginning in elementary school, Vito delved into in a wide range of readings and activities that regarded the humanities and history as well. In the summer of 1869, he returned to Ancona with his mother for a long visit with his Almagià cousins. During this stay, in addition to doing the schoolwork assigned for the summer holidays (a long-standing tradition in Italian schools, consisting even then in special reading assignments, essay writing, arithmetic exercises and word problems), always under his mother's watchful eye, he read several popular works of ancient history and literature. It was his mother who taught him French. This part of his youthful formation would make itself evident in later years in his book collecting, his attention to the history of science, his always wide-ranging and up-to-date

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[5]See Goodstein, *The Volterra Chronicles*, op. cit. The work by Volterra cited by Goodstein is 'L'évolution des idées fondamentales du calcul infinitésimal', in *Leçons sur les fonctions des lignes* edited by J. Pérès (Paris: Gauthier-Villars, 1913).

reading, as well as a particular interest in the world in general and in French culture in particular.

Having completed elementary school, the boy went on to attend lower secondary school at the Scuola Tecnica Dante Alighieri in Florence. It was during this part of his education that he began to tell his family that when he grew up he wanted to be a mathematician. It's a fact that at the age of 11 he read, on his own, Joseph Bertrand's *Arithmetique* and Adrien-Marie Legendre's *Géométrie*!

By the end of lower secondary school, the boy's intentions had begun to firm up. Some years earlier, in April 1870, his little family had undergone a change. His uncle Alfonso had married, and in a short time, after the birth of his children, the family grew to 9. Vito's determination to continue his studies (in the field of science) was a cause for inner satisfaction for his mother and uncle, but also a source of worry. The family situation seemed to call for Vito to begin work as soon as possible in order to contribute to expenses and ultimately become independent and assume responsibility for supporting his mother. However, a first compromise made it possible for him to enrol for 3 years in the Istituto Tecnico Galileo Galilei.

In the liberal period, this type of school was the forerunner, institutionally speaking, of that which under the Gentile educational reform of 1923 would be called the *liceo scientifico*, the upper level secondary school specialised in scientific subjects. In effect, the two decades that followed Italian unity saw the establishment of a great number of technical institutes in the larger cities in order to provide more opportunities for high quality technical education. The country was greatly in need of this, in order to achieve modern standards of living and foster industrial development. The technical institutes provided access to the scientific faculties in the universities and to schools of engineering. The network of technical instruction was placed under the supervision of the Ministry of Agriculture, Industry and Commerce, which was also responsible for a series of institutions at the university level.

This was the education itinerary that young Volterra looked to in order to follow his own vocation. In the meantime, he had begun to read the textbook for differential calculus by Bertrand, and to calculate centres of gravity and moments of inertia for various solids, dividing them into increasingly smaller parts and thus discovering intuitively the procedures underlying the operations of derivation and integration. Towards the end of the first year of secondary school, family pressure to leave his studies and begin to work began to mount. As so often happens, the only effect this had was to strengthen the boy's determination to finish the course he had begun. In autumn 1875, Vito was victorious in his cause thanks to an important ally: his uncle Edoardo, his mother's cousin. Born in 1841, Edoardo had studied in Florence and then enrolled in the University of Pisa. In 1861 he received his degree in applied mathematics and became a civil engineer, working first in the area of railways and later in that of construction. It was Edoardo who carried out the modernisation and enlargement of the port of Ancona. Beginning in 1867, he and some partners had formed their own firm and had won the contract to construct several stretches of railway in southern Italy. By the 1870s, building railroad lines and laying tracks in the south of the Italian peninsula had become his principal activity.

Edoardo was immediately aware of Vito's gifts and his determination, and he offered to contribute to ease the financial burden that his cousin's family had to bear to allow the aspiring mathematician to continue his studies. Uncle Alfonso was not in agreement. Seriously worried about family finances and his nephew's future, in summer 1867 he renewed his campaign to dissuade Vito from enrolling in the final year at the technical institute. At the same time, he attempted to convince Edoardo to hire the boy to work in his firm, or at least, to find him a job in the construction field. He had already been aware for some time that office work or a desk job would run too contrary to Vito's inclinations. Although he did not admit it openly to his cousins, Edoardo wanted Vito to finish his education. He thus adopted a delaying tactic, postponing Vito's hiring from month to month, and finally convincing Alfonso to allow him to complete the third year at the technical institute. Edoardo's personality would have a certain influence on the direction that Vito's mathematical studies would take: basically, his interest in applications of mathematics can be traced back to this period and to the advice of Edoardo Almagià.

Among other things, in the final year of his studies, one of Vito's professors was Cesare Arzelà (1847–1912), who had studied at the Scuola Normale Superiore of Pisa before holding the chair of algebra at the University of Palermo and that of infinitesimal calculus in Bologna. Arzelà is one of the 'historic' figures in the birth of functional analysis in Italy, and is still remembered today for the so-called Ascoli-Arzelà theorem. In a certain sense, the path taken by his research would develop in the opposite direction from that followed by Volterra: after having grappled with some questions of algebra and mathematical physics under the influence of Enrico Betti, Arzelà would dedicate himself to an in-depth examination of various topics of real analysis (inspired by the studies of Ulisse Dini) before meeting up with Volterra in the terrain of functional analysis. Their friendship would develop over the coming decades, but for the time being Vito was only a student in his final year at the technical institute, and Arzelà his professor, older by about 15 years. The depth of their friendship is immediately clear in the letter that Arzelà wrote to his friend on 15 July 1905: 'I can truly say that you are not only the most beautiful mind but also the most beautiful soul of all among our mathematicians'.

1.3   The University: 'Always Go Forward with the Same Tenacity of Purpose'!

It is easy to imagine how Vito's diploma and excellent scores, especially in mathematics, reopened family discussions about the young man's future. His uncle Alfonso continued to be worried and pessimistic about the possibility that, by enrolling in university, his nephew would effectively improve his prospects for employment.

It was at this point that the young man gained, in addition to his cousin Edoardo, another valuable ally: the physicist Antonio Ròiti (1843–1921), who had been Vito's professor at the technical institute and would become another friendly source

of support. Ròiti became a trusted and compassionate advisor, especially during in the years to come when, the young man's exceptional gifts having finally been acknowledged both by his family and by the academic world, it came time to guide his first steps towards a brilliant career. At this point, convinced of Vito's outstanding potential, Ròiti tried to convince Uncle Alfonso to spare no efforts in obtaining the funds to allow him to undertake studies at university: 'My dear sir, a portion of the funds assigned by the Province for places for students at the Central School in Paris are still available. The scholarship for your nephew Vito should be drawn from those funds. Today I will write to this purpose to the attorney Commendatore Niccolò Nobili, to advise him that you or your nephew will go see him'.[6] Obtaining the funds turned out to be more arduous than expected, in spite of the untiring efforts of the young but determined professor. He wrote once more to Alfonso, 'Most esteemed sir, I am writing to tell you that I have spoken with Commendatore Nobili who, if he finds it difficult to obtain a subsidy from the province for you nephew, hopes in any case to obtain it in some other fashion. To follow up on the steps he has taken, I recommend sending Vito to Cavalier Palagi, secretary to the Provincial delegation, along with the attached letter'. Ròiti did not stop at this suggestion: 'I had a letter written to Pisa [to ask] if there are places for students open there this year, and for your part you might speak with Professor Cesare Finzi, with my best regards. Finzi will probably be in the country, but I know that he comes to Florence quite often. You might ask him if Vito could apply for a Lavagna place. And do not forget to tell him that he distinguished himself in the diploma examinations. In the meantime, in order to know how to proceed in the measures I will take myself, I would like to know if Vito wants to dedicate himself to teaching or to engineering. In the first case he could perhaps continue his own studies in Florence at the Institute for Specialisation and obtain a scholarship in that way. But it is a good idea not to say anything about this to Finzi'. A few days later Vito told Ròiti about the outcome – not very encouraging, if not actually bad – of the attempts to obtain funds from the province and of Finzi's skepticism about the so-called 'Lavagna places', a kind of scholarship. Apart from everything else, he is still not sure what he wants to do in the future: '. . . if my studies could be continued in Florence, I would not hesitate to choose [the career] of teaching, because staying in this city where my family lives would be such a huge advantage; but in the case where I would pursue my studies at university, I would, if possible, put off a definitive decision until the end of the coming biennium'.

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[6]Ròiti's letter is dated September 1877. It is part of the collection of Volterra's correspondence housed in the 'Fondo Volterra' of the Accademia dei Lincei in Rome. Some of this correspondence has already been published. See in particular the large selection published in the exhibition catalogue *Vito Volterra e il suo tempo (1860–1940). Mostra storico-documentaria* edited by Giovanni Paoloni (Rome: Accademia Nazionale dei Lincei, 1990), which also contains the letter by Alfonso Almagià quoted earlier. Many of the other letters which will be quoted further along are published in their entirety in this catalogue.

By the beginning of October, the possibilities of continuing on after receiving the diploma at the end of the 3 years at the technical institute appeared to run out, along with Uncle Alfonso's patience. He wrote to his cousin Edoardo that the difficulties of obtaining financial aid or a scholarship had brought an end to the hypothesis that Vito might continue his studies, but that it's an ill wind that blows nobody any good, seeing as how the prospect of seeing him sacrifice himself for 5 or 6 years of hard study in a situation fraught with hardships only to find himself unemployed and forced to accept who knows what kind of job who knows where, was not all that attractive. Edoardo's reply risked provoking the ire of his cousin: he continued to be convinced that the best choice was that of leaving the young man free to continue his studies, and he renewed his offer of assistance. However, it was Ròiti who made the decisive move: just when Alfonso was about to hire his nephew to work at the Banca Nazionale, Ròiti offered him a job as assistant in his laboratory at the Istituto di Studi Superiori e di Perfezionamento, where he had a contract to teach physics. Vito thus enrolled in the first year of the Florentine school. He worked in the laboratory and, at the same time, began to develop a collaborative relationship which later grew into friendship with Ròiti. Most importantly, he gradually reached the decision to pursue his studies in Pisa. The tug-of-war over his future had lasted all the way from the end of lower secondary school!

The following year, in autumn 1878, he enrolled in the Faculty of Sciences at the University of Pisa. This was home to one of the finest schools of mathematics in the Kingdom of Italy. Pisa was also home to the Scuola Normale, which Vito dreamed of entering. Founded in 1810 during the Napoleonic Kingdom of Italy for the training of future teachers, it had been immediately closed after Napoleon's fall. When it was reopened in 1847, with Pisa then part of the Grand Duchy of Tuscany, its principal aim was still that of educating teachers, but in the first decades following Italian Unification, this original purpose came to be accompanied by another: that of becoming a centre for research – distinct from the university – and for training future researchers.

While the young Volterra was moving to Pisa, his two teachers at secondary school in Florence – Ròiti and Arzelà – managed to move into university careers. Each having passed competitive examinations, both were called to the University of Palermo, Ròiti to a chair in physics, Arzelà to chairs in algebra and analytical geometry.

In Pisa Volterra began to experience what it was like to be far from his family, and to gain some independence, although his progress was followed (sometimes a bit too closely) by Uncle Alfonso and his mother, who stayed in close contact. He could always count on the moral support of Ròiti, who never lost interest in his former student, even scolding him for having tried to pass directly to the second year of university and trying to take undue advantage of the last year of studies that he had spent in Florence. In December 1878 he wrote Vito: 'I had counselled you to not apply to the Rector of this university for registration in the second year . . . instead you laid yourself open to a refusal and now there is no remedy for it. But it is an ill wind that blows nobody any good, because as you quite aptly said, you now

have a chance to study Greek and Latin better and thus more easily obtain admittance to the Scuola Normale'.

Vito was actually quite worried about passing the examination in classical languages. In February 1879 Ròiti wrote to him: 'Give your all to your Latin and try to get rid of that in short order, and as soon as you have taken the exam, you'll be able to tell me something'. Ròiti then wrote to Alfonso to thank him for having helped him with a cash warrant, and added: 'Prof. Felici wrote me a couple of gratifying, because so well-deserved, lines about Vito. I am sure that, once he is past this terribly annoying exam in classical languages, he will go forward without further hindrances, and will have a brilliant career'. In May, Vito passed the exam. Ròiti wrote happily to him: 'Even more welcome was your second letter because I see that you have triumphantly overcome the most difficult trial you will ever face in the course of your studies. Bravo Volterra! Always go forward with the same tenacity of purpose, and the future will be yours'.

Thus, in the autumn the young Volterra was able to sit for the competitive examination for admission to the Scuola Normale, which he passed brilliantly, with the highest scores possible in each of the various sections. He was finally able to enrol in the second year of university, secure in the knowledge that as a student at the Scuola Normale he could count not only on a scholarship, but also on free room and board. Above all, he could count on access to one of the best research libraries in Italy, and on the opportunity for daily, constant contact with the best teachers at the Pisan university. During this period he would form friendships that would last his whole life, such as that with Carlo Somigliana (1860–1955), who like Volterra had been admitted to the Scuola Normale in 1879. Somigliana would become an esteemed mathematical physicist whose academic career would take him to teach at the universities in Pavia and Torino. Vito also became friends with students in the humanities, such as Guido Mazzoni, Carlo Picciola and Francesco Novati. Generally, however, he preferred the company of students of the sciences, who seemed to him to be more composed and serious. He found students of letters, with all their chattering, a little tiresome.

The rhythm of Vito's student life was cadenced by the schedule of his classes – Dini's classes in analysis were scheduled for 7:30 in the morning! – and by a program for studying that left very little time free. He sometimes complained in letters to his mother about this, and about how as a tutor he was gradually being assigned more and more work with the youngest students. At the end of the nineteenth century, Pisa didn't have much to offer, and as a smaller, provincial city, it began to struggle with a student population that was continually growing more numerous. A portrait of university student life in Pisa at that time is offered by the diary of a classmate of Volterra's, Rodolfo Bettazzi, who would later become the founder and first president of the Mathesis Association.[7] Bettazzi's background was different from Vito's, but they had a lot in common: he came from a deeply

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[7]The *Associazione Mathesis* was founded in 1895 as a national association of mathematics teachers.

Catholic family, but he had also come to Pisa from Florence, he had lost his father, and his family was certainly not very well-off financially. In his diary, he records how at the university 'in 4 years I missed only one class, and that was on the occasion of a visit by my grandfather to Pisa; and in all the examinations, both at the end of each year and the final examination for the degree, I always had the best possible scores with honours. In 4 years I never once left the house in the evening, except for a rare Sunday when I went to an old doctor, a family friend ... and one evening to the theatre to hear the phonograph, which was then a novelty'.

At the university, young Volterra attended the physics laboratory directed by Riccardo Felici (1819–1902) who, having earned his degree in Pisa in 1846, had taught there since 1862. 'I hope by this time', Ròiti wrote Vito in December 1878, 'Prof. Felici will have satisfied your request to attend his physics laboratory. But in any case I am sure that you will do me the kindness of giving him the letter attached here'. Thanks to Ròiti's introduction, Vito and Felici become close, as shown by Ròiti's writing to Volterra, 'Give my best to Felici, and tell him that I am completely in the dark about the competitive examination in Florence, because I have not yet seen notice of it in the *Gazzetta Ufficiale*. Still, at the beginning, the professor who most attracted Volterra's attention was Ulisse Dini. He found his lectures in analysis to be sometimes puzzling, and other times obscure, but always exciting. Dini gave his all, and never limited himself to merely repeating in class what were by then well-established notions. He illustrated the results of his own research for the students. In the end, although it was quite an effort to keep up with him, the concepts revealed themselves to be completely clear and concise in their formulation. Then, as he moved into to the third and fourth years, Volterra's interests moved mainly into the field of mathematical physics. At this point the instructor who became his primary point of reference was Enrico Betti. It was under Betti's advisement that Volterra earned his degree, on 30 June 1882.

1.4 A Rather Special Student

At 22 years old, Vito was certainly young, but all that young in a university system and a degree program that, as late as the first decades of the new century, would produce new graduates who had not even yet turned twenty. He had 'lost' a year in going from the technical institute in Florence to Pisa. But traces of his precocious talents are found in other contexts.

It was in 1881 – Volterra was still a student at the Scuola Normale – when his paper entitled 'Sui principii del calcolo integrale' (On the origins of integral calculus) was published in the *Giornale di matematiche*.[8] It was destined to become a solid point of reference for the development of the history of integration. Volterra

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[8]All of Volterra's publications are collected in the five volumes of the *Opere matematiche* published by the Accademia dei Lincei (1954–1962).

was also the only student who Betti mentioned specifically when, in summer 1881, as director of the Scuola Normale (which he directed until 1892, the year he died), he wrote his annual report to the Minister of Public Education: 'Mr Vito Volterra ... gives evidence of a singular aptitude for original research with two works, one of which, in mathematical physics, was judged worthy of being published in the *Nuovo Cimento*, the other, in higher analysis, was published in the *Giornale di Matematica* of Prof. Battaglini'. There were actually two papers on analysis by Volterra published in those years in the *Giornale di Matematiche*: 'Alcune osservazioni sulle funzioni punteggiate discontinue' (Some observations on pointwise discontinuous functions), dated February 1880, and that just mentioned, 'Sui principii del calcolo integrale', dated 21 April 1881, again as a student of the Scuola Normale.

To appreciate the significance of this, it is necessary to take a brief look at the history of the theory of integration. The definition of *definite integral* of a bounded function within an interval [a, b] by the German mathematician Bernhard Riemann – the one that is still today taught in calculus courses – dates to 1854. It spread around the years 1870–1880, thanks in part to the version of it provided by the French mathematician Gaston Darboux. With it, the divorce between the class of continuous functions and that of integral functions became final. These become two distinct classes, one of which contains the other: every continuous function is integrable, but the condition of integrability is no longer limited only to continuous functions. The operation of integration is also valid for discontinuous functions. But 'how' discontinous? The idea began to be take hold that the integrability of a function, that is, the possibility of calculating $\int_a^b f(x)dx$, depends on the set of its points of discontinuity and that, in any case, that set had to be 'small'. So, at most, *how many* points of discontinuity can a function 'afford' and still turn out to be integrable? *How* must these points be distributed within interval [a, b]? What must their *measure* be? Actually, each of these three questions tends to express the adjective *small* – used in describing the set of points of discontinuity of the function to be integrated – in a different way: *counting* the elements in the first case; giving a description of their arrangement in the second case (the *topological* version); and, finally, *measuring* the set itself in the third case. At that time it was not clear that these were three distinct formalisations, and there was a tendency to confuse them. The most advanced consideration seemed to consist in a theorem by the German mathematician Hermann Hankel, who would close the discussion with the statement that a function is integrable if and only if it is a *pointwise discontinuous* function, that is, if the set of points in which it is continuous is dense (in other words it can have an infinite number of points of discontinuity in the interval [a, b] as long as they are such that in each 'portion' of [a, b] there are points where it is continuous). The term *pointwise discontinuous* had been introduced precisely for the purpose of distinguishing these functions from those that are totally discontinuous, where the points of discontinuity do not succeed in forming a dense set.

It was here that Ulisse Dini (1845–1918) came in. Dini was Volterra's professor at the Scuola Normale, an esteemed scholar of real and complex analysis, trigonometric series, differential equations, and a true founder of the Italian school of

analysis. In Italy, he was the leading figure in the *rigourist* program in which German mathematicians would be particular involved, and which pursued the objective of rewriting, with the utmost possible precision and rigour, the basic notions of analysis (limits, derivatives, integrals, and so forth). In 1878, he published his *Fondamenti per la teorica delle funzioni di variabili reali* (Foundations for the theory of functions of real variables). This is how the program for rigorisation was described by Luigi Bianchi, a classmate of Volterra's and also a student of Dini's, who he would later succeed as director of the Scuola Normale: 'He himself tells us how, at the very the beginning of his scientific career, he had been struck by the observation that some fundamental principles of analysis appeared to be lacking, in the statements and the proofs, that absolute rigour that mathematics demands. How legitimate those doubts were is now known by all scholars of mathematics; but at the time, at least among us, no one had raised them, and it took the singular penetration of his intelligence to overcome the habit of taking for granted proofs that much of the time were only intuitive and recognise their insufficiency from the point of view of logic'.[9] Dini would go on to become the mayor of Pisa, a member of parliament and a senator of the Kingdom of Italy, and – from 1900 until his death – director of the Scuola Normale.

In the *Fondamenti* Dini had expressed serious doubts about the correctness of the proof given by Hankel of the theorem cited above, and in particular of the validity of the statement according to which every pointwise discontinuous function was necessarily integrable. He had not, however, been able to find a counterexample that would prove its falsity. The papers by Volterra provided the missing pieces of the puzzle, and are thus situated at a crucial junction of the theory of integration. In his first paper, Volterra gave an example of a set that is *nowhere dense* within an interval [*a*, *b*] – consider, for instance, the interval [*0*, *1*] of the infinite set *A* made up of the numbers 1, 1/2, 1/3, 1/4, . . .: no matter what interval in [*0*, *1*] is considered, it is always possible to find within it an interval that does not contain points of *A* – and that also does *not* have a null measure, since it cannot be enclosed within a finite number of intervals the sum of whose lengths is arbitrarily small. The three formalisations given earlier for the adjective small begin to distinguish themselves from one another. Volterra's set is 'small' from a topological point of view, but not from that of measure. Above all, this was the example that Dini was missing in order to prove the falsity of Hankel's statement beyond any possible doubt: 'Prof. Dini recognised that Hankel's proof was not rigorous; to this end it is possible to cite an example of a pointwise discontinuous function (which also has an analytic expression) which in a given interval is not capable of integration'. The same example is used in the second paper of 1881 to give an answer in the affirmative to another question, again posed by Dini: 'Most estimable Prof. Dini raised the doubt that . . . there exist functions whose derivatives are not capable of

[9]From Luigi Bianchi, 'In memoria di Ulisse Dini', published in vol. 1 of *Ulisse Dini Opere* (Rome: Cremonese, 1953).

integration'. Volterra effectively constructed a derivable function with a (bounded) derivative that was not integrable.

Volterra thus acquired two significant feathers for his cap. With the first example, he contributed to directing attention to the concept of a set of null measure, which was distinct from the topological concept of a set that is nowhere dense, and would turn out to be strategic in characterising the set of points of discontinuity of an integrable function. With the second, he showed that, generally speaking, the operations of derivation and integration are not inverses of each other. We are used to thinking of them like that, and so interpreting the so-called *fundamental theorems* of integral calculus. In particular, the second proves (under certain hypotheses) the equality $\int_a^x f'(t)dt = f(x) - f(a)$. In other words, if for a function f we first calculate the derivative f' and then the integral, the two operations neutralise each other – they are exactly the inverse of each other – and we return to the original function f. Volterra's counterexample has the merit of underlining the need for the hypotheses that make this theorem true. Otherwise, it cannot be taken for granted that it is possible to derive a function and then 'go back', that is, integrate its derivative (since the derivative may not be integrable).

The paper of 1881 also introduced the notions of *upper and lower derivatives* and their relative symbols. In the case of upper derivatives, Volterra also proved that it is 'impossible to find a function that is both discontinuous at all irrational points and continuous at the rational ones', and thus there cannot exist a continuous function that associates irrational numbers to rational ones and vice versa. With his critical examination of the links between derivation and integration (that is, Riemannian integration), Volterra staked a claim for himself within Dini's program for rigorisation. Later, as we said, in his second 2 years of university studies his attention would move increasingly towards the field of mathematical physics and the research interests of Enrico Betti (1823–1892), not least because it was so difficult to meet with Dini, who was at that time dividing his time between Pisa and Rome in order to carry out his responsibilities in the public sector.

Author of important works on potential theory and elasticity, Betti also made significant contributions in algebraic topology and the study of elliptic functions and complex variables. His meeting and subsequent friendship with Riemann, who came to Italy for health reasons and remained in Pisa from 1863–1865, were decisive for his mathematical personality. Volterra would always credit Betti with being one of the founders of Italian mathematics after Unification. Let's listen once again to Luigi Bianchi, who compared the different mathematical sensitivities of Betti and Dini: '... as little worthy as I was of it, it became a real intellectual pleasure to attend the lessons of these two great *maestri* of ours, who were as different in their ways of exposition as they were in the nature of their mathematical minds. Adopting the views expounded by Henri Poincaré ... these differences could be expressed by assigning Betti, without any hesitation, to the rank of 'intuitive' mathematicians, and Dini instead to that of the 'logicians'; both of them, however, could justifiably be collocated among the mathematical "inventors"'.[10]

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[10]Bianchi, 'In memoria di Ulisse Dini', op. cit.

Chapter 2
Professor in Pisa

2.1 Just Graduated and Teaching Right Away

Young Vito, student at the Scuola Normale in Pisa, graduated in physics in 1882 with a thesis on hydrodynamics written under the advisement of Enrico Betti. Other graduates from the Scuola Normale in the 1880s included Salvatore Pincherle, Gregorio Ricci-Curbastro, Luigi Bianchi, Carlo Somigliana and Mario Pieri, and later, in the 1890s, Federigo Enriques, Gaetano Scorza and Guido Fubini. Each one of these is a 'luminary' in the history of mathematics, and they provide excellent testimony to the level of the Scuola Normale. In the previous chapter, we mentioned the teaching faculty that Volterra found when he enrolled in the Scuola Normale. Above all, it was Dini and Betti who provided the orientation for his early choices. They also contributed to making Pisa a major centre for Italian mathematics, which, at the beginning of the twentieth century, would generally be acknowledged as third in the ideal ranking of mathematics by country, after France and Germany. This is a significant achievement for a mathematics that had essentially begun from scratch, in 1860, at the time of Italian unification!

Before following the young graduate in his first steps in the world of research, it is helpful to get a perspective on the situation: we are following the tracks of one who will become a leading figure in the great *leap forward* that will bring Italian mathematics close to the traditional excellence of the French and Germans, in the attempt – and here the outcomes will reveal themselves to be much more problematic – to orient the cultural and social development of the country itself. At the turn of the twentieth century there was a boom in all of the disciples in which mathematical research was traditionally divided (and in which we find many of the 'Pisan' names just mentioned): *analysis*, with the studies of Brioschi, Dini, Casorati, Arzelà, Ascoli, Peano, Pincherle, Vitali, Tonelli, Fubini, and others; *geometry*, with the contributions of Cremona, Battaglini, Bertini, Segre, Fano, Beltrami, Castelnuovo, Enriques and Severi in particular, and all members of the Italian school of algebraic geometry in general; *mathematical physics*, with Betti, Ricci-Curbastro, Bianchi, Somigliana, Levi-Civita and others. In short, Volterra

had the great good luck to find himself in the *right place* at the *right time*, and he also deserves credit for having made a decisive contribution to that 'right time'. His contributions regarded analysis, mathematical physics and other fields. His research work is in any case strongly unified by several constants that are easily perceived in his thoughts and in his actions. These constants are also very significant.

It was under Betti's advisement, as we have seen, that Volterra earned his degree in 1882, and it was also with Betti that, in the month of December, immediately after receiving his degree, he began his academic career, as his assistant: 'Dear Volterra, yesterday the faculty resolved to propose to the Minister that he anticipate the opening of the competition for the chair in rational mechanics, and that in the meantime I teach these classes until the professor who wins the competition takes over the chair. I accepted this responsibility, but I asked that the government be requested to give me an assistant, to whom will be paid a salary of 1,250 lire; and the Faculty also presented this proposal to the Minister. I have reason to believe that the Faculty's proposal will be accepted. As soon as approval is given, I will have to propose an assistant, and I would like it to be you. Please write me if you are willing to accept'.

Volterra's family, his mother Angelica and his uncle Alfonso, continued to stay in close contact with the young graduate even during these first steps in his adult life, and followed his progress with loving concern. Antonio Ròiti , his 'old' professor of physics at the Istituto Tecnico 'Galileo Galilei' in Florence, by now a professor at the University of Palermo, continued to advise him – even bluntly, when necessary – so that he doesn't let good opportunities pass him by. With his nomination as Betti's assistant, Volterra's career accelerated unexpectedly. The following year, in spite of the initial cautiousness of his 'boss', he sat for and passed the competitive examination for the chair of rational mechanics that Betti had mentioned in the letter quoted above, in the same University of Pisa where he had earned his degree only a few months earlier!

Volterra taught for 10 years in Pisa, holding the courses in rational mechanics and graphic statics (as well as mathematical physics, after Betti's death). For a short while he would also be asked to work in the library of the Scuola Normale. These were important years for Pisa as well as its university. It is easy to see how they would be decisive for a university professor who was not yet 30 years old!

Of average height and build for that period (just under 170 cm tall), with dark brown hair, he began to grow a beard. At this point in time he didn't express any particular political opinions, although it appears from some phrases in his letters to his family that he tended towards a moderately conservative stance, and supported the monarchy. Of course, this was the prevailing political climate in the Scuola Normale where he had been trained. When Dini told Betti about the positive outcome of the administrative elections in a Tuscan college in 1882, he wrote that 'we have even conquered the citadel of progressivism',[1] a joking way of summing up the political leanings held by the group of mathematicians. In his

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[1]The letter is quoted in M. Berengo, *Cultura e instituzioni nell'Ottocento italiano* (Bologna: Il Mulino, 2004).

first contacts with students, *Professor* Volterra showed himself to be particularly severe and demanding. All things considered, however, he still moved with a certain reserve and uneasiness in the academic world. Starting in 1887 he lived with his mother, who left Florence definitively to come to be with him. Those who knew him best spoke of him as an extremely cordial young man who was immediately likeable. Ernesto Pascal (1865–1940), almost the same age as Volterra, and like him, destined to teach mathematics, first at the University of Pavia, and then in that of Naples, had this to say about him: 'Prof. Volterra is an angelic young man, of characteristic modesty'.[2]

The extent of his talents began to recognised beyond the confines of Pisa. Gradually the scientific and academic worlds in Italy opened their doors to him. In 1887 he was awarded the gold medal for mathematics by the Società dei XL, the forerunner of the Italian National Academy of Sciences. The following year he was nominated *corresponding member* of the Accademia dei Lincei (he would be elected as an even more prestigious *national member* in 1899). In 1891, he became a member of the Palermo's Circolo Matematico, and a Knight of the Order of the Crown of Italy. In 1892 – at the death of Betti – he was elected dean of the Faculty of Sciences of the University of Pisa. He also succeeded his mentor in the direction of the journal *Nuovo Cimento*.

2.2 Scientific Work During the Period in Pisa

The years in Pisa following his winning the chair in rational mechanics were very important for Volterra from the point of view of his research as well as the development of his scientific personality. First he had worked in analysis with Dini, and then in mathematical physics with Betti. In his case (but also, in actual fact, that of Betti) it is difficult to draw a clear line between the works in analysis and those in mathematical physics. We will attempt in any case to do this – in order to provide an orientation within a body of work that is not only vast, but also suggests right away several different ways in which it can be read – but it must be noted that this distinction has to be applied with a certain degree of caution, in part because in the articles of 'physics' the use of instruments of analysis is far from marginal, and in part because those we will call 'analytical' very often comprise points of view, examples and applications drawn from physics.

In any case, the various works – whether of analysis or of mathematical physics – are never reduced to a mere proof of a theorem. Rather, theorems, corollaries and observations are used as the means for developing a genuine scientific discourse.

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[2]The quote appears in a letter from Pascal to the mathematics historian Federico Amodeo, dated 23 November 1887, published in *Dalla "Moderna Geometria" alla "Nuova Geometria Italiana" Viaggiando per Napoli, Torino e dintorni. Lettere Di: Sannia, Segre, Peano, Castelnuovo, D'Ovidio, Del Pezzo, Pascal e Altri a Federico Amodeo*, Franco and Nicla Palladino, eds. (Florence: Olshki, 2006).

Sometimes the investigations almost necessarily include long prefaces or theoretical 'parentheses' in order to construct the instruments for their formalisation; other times they are more absorbed in the possibility of solving new problems in physics or making a contribution to the clarification of older experimental observations. In presenting some of Volterra's research works, we will have several opportunities to underline their strong unitary character. For the moment we will limit ourselves to providing an example of the style he adopted, by quoting a passage from 'Sopra alcune condizioni caratteristiche delle funzioni di una variabile complessa' (On some characteristic conditions of functions of a complex variable). This long, involved paper – about 50 pages! – dated 12 May 1882 includes a note saying that the author is a student at the Scuola Normale. In the introduction to this work of – according to us – analysis, we read that 'the present paper provides a solution to the problem of the determination of functions of complex variables defined under certain conditions in finite fields. These solutions lead to the integration of the differential equation $\Delta^2\mu = 0$ with given boundary conditions, as can be expected due to the connections between the two problems. It should be noted that the formulae found solve an equal number of questions of physics relative to the distribution of temperature and constant galvanic currents'.

For the time being, let's set aside this unifying aspect and for the sake of convenience adopt a more rigid classification. Volterra's first paper – the one that Betti would write about in his report to the Minister of Public Education, published in the *Nuovo Cimento* – dealt with the calculation of the potential of an ellipsoid. In all, during the period in Pisa and up to the beginning of the 1890s, Volterra would publish no fewer than about 20 (!) articles on mathematical physics. This output is mainly concentrated between 1882 and 1885, and in the final years of his time in Pisa. It goes from potential theory to the first observations of elasticity, passing through various questions of hydrodynamics, electrochemistry (suggested to him by Antonio Ròiti), mechanics, optics, electrostatics and electrodynamics (with particular emphasis on analytical aspects and relations to the calculus of variations).

The most important of these studies, 'Sur les vibrations dans les milieux birefringents' (On vibrations in birefringent media), was published in 1891 in *Acta Mathematica*, the journal published by the Swedish mathematician Gösta Mittag-Leffler (1846–1927), and this time it wasn't preceded by a earlier paper on the topic published with the Lincei. It was concerned with analysing the mathematical laws of the propagation of light in doubly refracting materials and substances, and their property of decomposing incident rays into two polarised rays that vibrate in planes that are perpendicular to each other.

The problem had a long history. It had already been addressed by Christiaan Huygens in his 1690 *Traité de la lumière*. Later it attracted the attention of physicists and mathematicians such as Fresnel, Hamilton, Plücker, Navier, Cauchy, Green and Stokes, with an increasing focus on the model provided by the theory of elasticity.[3]

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[3]See the essay by Lars Gårding, 'History of the Mathematics of Double Refraction', *Archive for History of Exact Sciences* 40, 4 (1989): pp. 355–385.

In 1860, the French mathematician and engineer Gabriel Lamé published a collection of his lectures at the Sorbonne in a volume entitled *Théorie mathématique de l'elasticité des corps solides*. This is what Volterra had to say: 'Lamé dedicated the 22nd and 23rd of these lessons to the mathematical theory of elasticity in search of a single centre of disturbance in the propagation of light in birefringent media'. In order to study the phenomenon from a mathematical point of view, Lamé formulated a system of partial differential equations for which he found special solutions. These formed the basis for the consideration of the Russian mathematician Sophie Kowalevski, who also relied on a method used earlier by Weierstrass to solve simpler systems. The aim was to get beyond some of the physical inconsistencies in Lamé's reasoning, while at the same time obtaining the general solution to the system. Her research was published in 1885 in *Acta Mathematica*. Mittag-Leffler had been able to bring her academic wandering to an end, by finding her a teaching position (and later a chair) at the University of Stockholm. Even the Swedish intellectuals had to struggle to accept the fact that a woman could dedicate herself to scientific research. The playwright August Strindberg was outraged over her appointment, considering a woman professor of mathematics to be a 'monstrous' phenomenon, 'pernicious, useless, and unpleasant'.

Sophie Kowalevski (1850–1891) was the first woman in the world to receive a doctorate in mathematics and to enjoy significant fame in the scientific world. To find another woman mathematician of her calibre you have to go back to Maria Gaetana Agnesi (1715–1799). Of Russian nationality, in order to be able to continue her studies and be granted a passport, she 'invented' a marriage of convenience. This is how she was able to study in Germany, where in 1874 she earned her doctorate under the advisement of Karl Weierstrass. Earlier she had been in Paris, during the time of the Paris Comune. Her political ideas were not very 'orthodox', and this did not help improve her situation, nor her prospects for an academic career. Luckily, as mentioned, she found the support of mathematicians such as Weierstrass and Mittag-Leffler. In 1888, she was awarded the *Prix Bordin* of the French Academy of Science. It would not be until 1908 that another woman – Marie Curie – obtained a university chair.

Going back to Volterra, this explains why he published his work in *Acta Mathematica*, because it is the journal that published the work of Kowalevski. Volterra's contribution grew out of a critical examination of the procedure followed by Lamè. The transformation of his solution into another form made it possible to uncover an error that even Kowalevski had missed, and which thus nullified her search for the general solution: 'This property [of being multi-valued functions] is hardly perceived at first glance, when one examines these quantities in the form that Lamé had given them. This is why he was mistaken when he believed that they could represent the luminous vibrations coming from a centre of disturbance. The same functions appear in the paper of Mme. Kovalevskii. When one sees that they are multi-valued functions, one also sees that the method discovered by Weierstrass for the integration of the partial differential equations can be used for integrating the Lamé equations by using the coordinates of Weber'. On 17 April 1892, he wrote in more explicit terms to the French physicist and philosopher of science Pierre

Duhem: 'Some time ago I began research in the electromagnetic theory of light, but I have had to interrupt my work because I recognised, to my great surprise, that the integrals relative to double refraction given by Lamé and Mme. Kowalevski were affected by the same analytical error, although the starting points for the geometers who have studied this question were different'.

As was the case in mathematical physics, in the period Volterra spent in Pisa, his publications concerning analysis numbered about 20! After his papers of 1881, which we discussed in the previous chapter, Volterra never again addressed questions related to the foundations of the discipline, such as the relationships between derivation and integration. This was either a fortunate choice, or happy intuition, given that both the interest in rigorisation of real analysis and the moment of Dini's greatest influence were already past their prime. It was Betti who, with increasing frequency, presented the articles of his former student and by then young colleague to the Academica dei Lincei.

The problems concerned regarded above all the study of functions of complex variables and differential equations (ordinary and partial). In 1890 there appeared two articles on the calculus of variations. In 'Sopra un'estensione della teoria di Jacobi-Hamilton del calcolo delle variazioni' (On an extension of the Hamilton-Jacobi theory of the calculus of variations), Volterra generalised to double integrals the (partial) differential equations of Hamilton-Jacobi, which at the time had only been used in the so-called 'simpler problem' (which deals with determining the function $f(x)$ that makes the quantity $J = \int_a^b f(y, y, y')dx$ minimum or maximum), considering J as a function of a, b, $y(a)$, $y(b)$. For Volterra, this provided another opportunity to use his new theory of functions of lines: 'If one goes from simple integrals to the case of double integrals, we have one or more lines forming the boundary of the field of integration and on this must lie the arbitrary values of the unknown function'. With the functions of lines, we arrive to the most significant research of Volterra's years in Pisa. Functional analysis was invented by Volterra – a young man just over 25 years old! – with two papers published in 1887 entitled 'Sopra le funzioni che dipendono da altre funzioni' (On functions that depend on other functions) and 'Sopre le funzioni dependenti da linee' (On functions that depend on lines), which were followed by others – very closely related – in which the focus was moved to certain questions of complex analysis. These included 'Sopra un'estensione della teoria di Riemann nelle funzioni di variabili complesse' (On an extension of Riemann's theory of functions of complex variables), 'Sur une généralisation de la théorie des functions d'une variable imaginaire' (On a generalisation of the theory of functions of an imaginary variable, published in *Acta Mathematica*, Mittag-Leffler's journal) and 'Delle variabili complesse negli iperspazi' (On complex variables in hyperspaces).

The term *functional analysis* refers to the study of functionals (and later, their spaces). At that time Volterra did not yet use term, which would be coined in 1903 by Jacques Hadamard (1865–1963), one of the most important mathematicians of the first half of the twentieth century, and another of Volterra's close French friends. He preferred to use the terms that appear in the titles of his papers: 'functions that depend on other functions' and 'functions that depend on lines'. The meaning is the same, even though the term 'functional' would be adopted by

later generations, and would become the definitive one. What is a functional, or a function that depends on lines?

The generalisation of the concept of function is simple. While we use the term *real function of a real variable* to indicate a correspondence that associates a real number to another real number, term functional is used to indicate a correspondence between any given set and a real number. In Volterra's terminology, which was initially less general, it is a correspondence which associates a real number to an element of the set of continuous functions or of the curve that represent them geometrically. He thus talks about a function that depends on another function – a concept that must must not be confused with that of a composite function – or a function that depends on a line. The generalisation involves the independent variable of the correspondence, not the dependent variable. In the passage from functions to functionals, everything that has been said regarding the behaviour of the independent variable (for example, its approaching a limit value) will have to be reformulated, while those that involve the dependent variable can be 'copied' without qualms because – from this point of view – nothing has changed.

For mathematicians, ideas for new research can come from previous studies – these are the so-called *internal* motivations – where further reflection is aimed at making concepts more precise and extending them. Ideas for research, however, can also come from observing nature in the real world – the so-called *external* motivations. Of course, as far as mathematics is concerned, especially modern mathematics, the idea of 'real world' is understood in an indirect sense. There might be cases where a mathematician studies planetary motion or a particular upheaval in the stock market and is able right away to formulate a new theorem, but more often it happens that the echoes of the real world filter in from others – astronomers (for planetary motion) or economists (for the stock market) – and that the mathematician actually works with a world that is already on paper, that is, the world of astronomers or economists. Both internal and external motivations were at play in the generalisation performed by Volterra with functionals, in a way that was something of a paradigm. In the paper published in *Acta Mathematica* cited above, he wrote: 'These functions arise in a number of questions in physics. They can also be related to analytical questions'. Later he would reaffirm this: 'I realised the necessity of considering the functions of lines because many natural phenomena lead to the study of quantities that depend on an infinite number of variables. Many problems of analysis also lead to the same quantities'.[4]

Volterra was driven to invent the theory of functionals by the ascertainment that expressions that depend on other functions are present in many analytical developments, such as, for example, the soluion of partial differential equations (where an integral comes to depend on one or more abitrary functions) or in complex analysis. In fact, one of the initial motivations was that he expected to be able to apply the new theory successfully to some studies of complex analysis:

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[4]Vito Volterra, 'Funzioni di linee, equazioni integrali e integro-differenziabilii', *Anales de la Sociadad Cientifica Argentina*, 1921.

'I allow myself to mention in this paper some of the considerations that serve to clarify the concepts I believe it necessary to introduce in order to extend Riemann's theory on functions of complex variables, which I think can be used to good advantage in various other research projects'.[5] In the same year, 1877, Poincaré proved that the integral of a function of two complex variables, extended to a surface contained in the domain set of its boundary, turns out to depend only on this. Volterra observed: 'Poincaré, in generalizing Cauchy's theorem, has shown that the integral of a uniform function of two imaginary variables taken over a closed surface is zero. It can be inferred from this that, when the surface of integration is not closed, the integral depends on the lines that form the contour of the surface. So we see that the integration of functions of two variables leads to functions of the lines'.[6]

These are internal motivations. The external references are to the 'many experiments in physics and mechanics' where likewise, spontaneously, the concept of function can be glimpsed: 'for example, the temperature at a point in a thin plate conductor depends on all the values of the temperature at the edges; the infinitesimal displacement of a point on a non-extendable flexible surface depends on all of the components of displacement of the points on the edges, parallel to a given direction'. Even the geometric version of functionals, considered in the work on the set of closed curves in a three-dimensional space, is familiar to the physicist, given that 'it appears spontaneously when one thinks of certain electrical phenomena. Think of an electric current that flows in a closed linear circuit with an intensity equal to I and that is located in a magnetic field. The potential energy of the current will depend only on the shape, the position of the circuit and on the direction in which the current flows; thus to each closed line that is traced in the magnetic field followed in a given direction, there will correspond a value of potential energy'.

The originality of the internal motivations did not escape Hadamard, who underlined the almost aesthetic value of the theory formulated by Volterra: 'much more surprising is the fate of the extension given to that initial conception in the last part of the nineteenth century, chiefly under the powerful impulse of Volterra. Why was the great Italian geometer lead to work on functions in the same way in which infinitesimal calculus had worked on numbers, that is, to consider a function as a continuously variable element? Only because he realised that it this was a harmonious way of completing the architecture of the mathematical building, just as the architect sees that the building would be better poised by the addition of a new wing'. The French mathematician did not mention what we have called external motivations, but he cannot help but underline the unexpected applicability of the new concepts: 'that 'functionals', as we called the new conception, could be in direct relation with reality, could not be thought of otherwise as mere absurdity.

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[5]Vito Volterra, 'Sopra le funzioni che dipendono da altre funzioni', *Rendiconti Accademia dei Lincei* 4, 3 (1887): pp. 97–105, 141–146 and 153–158.

[6]Vito Volterra, 'Sur une généralisation de la théorie des fonctions d'une variable imaginaire', *Acta Mathematica* 12 (1889): pp. 233–286.

Functionals seemed to be an essentially and completely abstract concept of mathematicians. Now, precisely the absurd has happened'.[7]

Once the concept of functionals had been introduced, in order to work with this new mathematical reality it was necessary to construct a *calculus* that was similar to the classic one known to analysis for functions of a real variable: the notion of limit, the definition and the calculation of derivatives, and so forth. The functional analysis of the twentieth century, beginning with Fréchet's doctoral thesis of 1906,[8] studied functionals as definite correspondences on a given set and tended to privilege the study of minimal structures that had to be introduced into that set in order to develop the new calculus. One of the first definitions regarded the *metric structure*, which made it possible to specify the concept of *neighbourhood* and consider the operations of limit (and thus of derivative as well).

None of this is yet present in Volterra. He had no need for an abstraction 'pushed' this far, and for him the motivations behind his studies remained essential for determining the level of abstraction. Volterra's functions of lines – it is no coincidence that the terminology is also different – are definite correspondences to a 'concrete' set and are in any case specific, such as that constituted of functions that are continuous in an interval. There is no general definition of metric space. The definition of derivative itself still refers to the limit of a real parameter (and to a procedure that is familiar to all those who work with the calculus of variations): considering an initial function f_0 and the corresponding value of the functional U (f_0), the incremented function is written as $f_0 + tg$ where t is a real parameter and g is a pre-established function that continues to give, as t tends to 0, the 'direction' of the increment. The limit of $[U(f_0 + tg) - U(f_0)]/t$ for t that tends to 0, is the first derivative of the functional with respect to direction g.

The introduction to his 1887 paper is not written in the terse, unadorned style that we are accustomed to today, especially when the definitions are given. At the end of the nineteenth century, the habit of 'hiding' the motivations and the mental structures that led to the introduction of a certain definition was not widespread among mathematicians. Nor does Volterra spend much time examining the primary properties of the derivative of a functional. Instead, he is concerned with calculating the expressions of the different variations of a functional (first, second, …nth) because their consequential development with *Taylor's formula* serves him for the analysis that will follow. It is the applications – including applications in mathematics, to be sure – that dictate the course for the procedure. The point of reference is research as a whole, not passion for in-depth study as an end in itself.

We spoke earlier of the paradigms that are constantly present in Volterra's work. The first of these that we have the chance to underline is that of the juxtaposition and interweaving of 'pure' research and research that is in some way applied, of

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[7]Jacques Hadamard, *The Mathematician's Mind: Psychology of Invention in the Mathematical Field* (Princeton: Princeton University Press, 1996), p. 130 (1st ed., 1945).

[8]Maurice Fréchet, 'Sur quelques points du calcul fonctionnel', *Rendiconti del Circolo Matematico di Palermo* 22, 1 (1906): pp. 1–72.

analysis and mathematical physics, of internal and external motivations. This leads us naturally to acknowledge the particular importance of applications, and the role that the instance of application plays in identifying the most suitable level of abstractions. We will see some aspects of this come into play a few decades later in a controversy that involved Volterra and the French mathematician Fréchet (1878–1973) regarding precisely the definition of derivative of a functional. Fréchet, author of important contributions to analysis and probability, is considered one of the founders of functional analysis. We have already mentioned his doctoral thesis of 1906, which dealt with the study of metric spaces.

Volterra's definition of the derivative of a functional – later referred to in the literature as the Gâteaux derivative[9] – generalises the usual concept of directional derivative in the choice of the increment tg and its progressive reduction only by means of the scalar t, which, tending towards 0, reduces the incremental but leaves its form in some ways unaltered. Later, Fréchet introduced a definition of the differential of a funtional that generalises the usual concept of total differential for functions of n variables. The difference between the two procedures was immediately evident when Fréchet wrote the incremented function as $f_0 + g$ and evaluated the validity of the linear approximation of $[U(f_0 + g) - U(f_0)]$ in the whole neighbourhood of f_0 by means of the limit for g tending to 0. It is easy to compare the two definitions: that of Fréchet considers a generic increment g, making it go to 0 regardless of how it does so, and provides a more precise identity for the class of functionals – those that are 'Fréchet differentiable' – destined to play a fundamental role in *calculus*. A functional that is Fréchet differentiable is also Gâteaux differentiable, while the inverse is not true (Fig. 2.1).

Behind the two different definitions there are also a different ways of thinking about the concept of generalisation and the appropriate level of abstraction. In 1965, Fréchet wrote to another famous French mathematician, Paul Lévy: 'While I think that Volterra achieved a great step forward in giving at least one definition of the differential of a function whose argument is a function, on the other hand, I think that the definition is bad. I have not applied his definition . . . it is a definition entirely different from that of Volterra'. Earlier, in somewhat more diplomatic terms, he had written that 'the traditional method, due to Lagrange, is not to treat the functions of lines similarly to the functions of numbers, but to pass through functions calculated on a family of lines depending on a parameter and which become none other than functions of the numeric parameter. This is also the method adopted by Volterra for more general functions (of lines) studied by functional analysis. We believe that it is better to get to the bottom of things and avoid problems by abandoning the intermediate parameter and taking the line directly as an absolutely independent variable'.

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[9]On the reasons that led – at least on the level of terminology – to the collocation of Volterra's contribution in second place, see one of Angelo Guerraggio's two 'updates' of Carl Boyers classic text, *Storia del Calcolo* (Milan: Bruno Mondadori, 2007).

Fig 2.1 Maurice Fréchet in 1910

Volterra did not discuss the terms of the disagreement, but he never missed an opportunity to reiterate that the development of mathematical thought cannot be guided only by aesthetic or formal criteria, by the elegance of a construction or its greatest generality. Addressing Fréchet directly in a letter of 17 November 1913, he wrote that 'naturally I had before me at that time, in 1887, a number of problems (integral equations, equations with functional derivatives, etc.) so that I could not stop for what from my point of view were issues secondary to the general applications of concepts I had laid out'.

2.3 The First Trips Abroad

The results we have discussed were soon well known even beyond Italy's borders. We have already mentioned the first honours Volterra received in Italy. He also received – as early as the second half of the 1880s – similar honours in the international arena, beginning with France, which it is no exaggeration to say was always thought of by Volterra as a second homeland.

In 1888 Volterra became a member of the *Société Mathématique de France*. This happened in the same year as his first trip abroad. In Pisa, while still a student, he had met the Swedish mathematician Gösta Mittag-Leffler, whom we have already come across in relation to Sophie Kowalevski. That meeting gave rise to reciprocal esteem – even on the part of the already established Swedish professor for the young Italian student – which over the course of the years turned into a lifelong friendship. Mittag-Leffler died in 1927. He was known for his work on

Fig. 2.2 Henri Poincaré

differential equations and on functions of a complex variable, and for the very active role he played in the international mathematics community, especially as a citizen of a neutral country in the aftermath of dramatic conflicts, such as the Franco-Prussian war of 1870 and World War I. In 1882, at the suggestion of the Norwegian mathematician Sophus Lie, he had founded the international journal entitled *Acta Mathematica*. With Mittag-Leffler, Volterra visited Switzerland, and had the chance to meet important, esteemed mathematicians like Karl Weierstrass, Georg Cantor, Sophie Kowalevski, Hermann Schwarz and Adolf Hurwitz. Later, during the Easter holidays of that same year, 1888, he travelled to Paris, where he met Poincaré. Poincaré invited him to come again the following year to the French capital to take part in the *Congrés international de bibliographie des sciences mathématique*, organised in concomitance with the Exposition Universelle of 1889. This was the World's Fair that celebrated the centenary of the French Revolution, and was highlighted by the inauguration of the Eiffel Tower! Where once there had been few opportunities for leading figures of the various national communities to meet, the idea for the first international congresses (Zurich 1897 and Paris 1900) was 'in the air'. Volterra attended the International Congress for Bibliography of Mathematical Science together with Giovan Battista Guccia, who had shortly before, in 1884, founded the Circolo Matematico of Palermo, which would soon enjoy significant international renown and esteem (Fig. 2.2).

Letters from these years show how Volterra had by that time entered into a 'circle' of correspondence and acquaintances with figures who were particularly

esteemed and knowledgeable. It was, however, the meeting with Poincaré that would be decisive, much more than a mere acquaintance with a foreign colleague that made it possible to intensify the network of scientific contacts. First of all, there was the stature of the correspondent, whose family was destined to leave their mark on the history of France (and beyond): Raymond Poincaré, several times prime minister and president of the Republic during the years of World War I, was his cousin. Apart from this, in the decades at the turn of the century, Henri Poincaré (1854–1912) – mathematician, physicist, philosopher of science – was undoubtedly one of the leading figures of world science. In the year of the World's Fair and the *Congrés international*, his reputation took a great leap forward. In 1889 he won the competition established by Oscar II, King of Sweden and Norway, in honour of both his sixtieth birthday and of his royal highness's interest in mathematical progress. His paper, 'Sur le probléme de trois corps et les équations de la dynamique' (On the three-body problem and the equations of dynamics) was soon followed by his three-volume *Les methodes nouvelle de la mécanique celeste* (*New Methods of Celestial Mechanics*). It was precisely the study of dynamic systems in the context of celestial mechanics and their stability that gave rise to what we now call the theory of *deterministic chaos*. The germ was provided by an observation on the sensitivity of the evolution of the dynamic system with respect to a slight perturbation of the initial conditions: 'it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible'.[10] Poincaré created new mathematics without worrying about disciplinary confines or traditional distinctions between pure and applied mathematics, driven as he was by thirst for scientific comprehension of natural phenomena. *Pure* and *applied* were categories that were too narrow for him. He was perhaps the last 'universal' mathematician in history. His contributions ranged from mathematical physics (celestial mechanics, fluid mechanics, relativity, cosmology, optics, electricity, etc.) to the study of functions of complex variables, the qualitative theory of differential equations, non-Euclidean geometries, algebraic topology and number theory.

Volterra was particularly struck by this broadly encompassing way of conceiving and practising science. He was fascinated by the constant concern for uniting the rigour of calculus with the necessity of understanding physical reality. It was no coincidence that he repeatedly proposed Poincaré for the Nobel Prize in physics. Instead, he was called to commemorate him at Rice Institute in Houston on 10 October 1912, a few months after his death. On that occasion he recalled that for him Poincaré had been both a *maestro* and a constant source of inspiration. Volterra's sincere appreciation for Poincaré's genius and the contributions he made are evident throughout the discourse, as are the fact that he shared many of the choices that accompanied the investigations of the French mathematician:

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[10]Henri Poincaré, *Science and Method*, Francis Maitland, trans. (London: Thomas Nelson and Son, 1914), p. 68.

a mathematics at once both pure and applied, not one domesticated within rigid, predetermined classifications, one deeply and formally involved, never in any case reduced to a simple logical development of formal rules, and in which intuition and experience continued to play essential roles. Volterra wrote: 'During these last 30 years there has been no new question, connected even remotely with mathematics, which he did not subject to his deep and delicate analysis, and enrich with some discovery or fruitful point of view. I believe that no scientist so much as he lived in constant and intimate relation with the scientific world that surrounded him. . . . That is why, if we were to characterize the recent period of the history of mathematics by a single name, we should all give that of Poincaré, for he has been without doubt the most widely known and celebrated mathematician of recent years. . . . There is certainly a philosophy that is Poincaré's, and an analysis, a mathematical physics and a mechanics that are Poincaré's, which science can never forget. His renown during his life was great. Few scientists and a very few mathematicians have had celebrity equal to his. A physicist would find the reason for this in what I have just been saying, remarking that his spirit and the spirit of his time vibrated in unison, and that he was in phase with the universal vibration'.[11]

To return to our discussion of trips abroad, because in addition to French, Volterra knew German (though little English), he took advantage of that to plan a month of study in Germany. His destination was the University of Göttingen, where mathematicians of the calibre of Dirichlet and Riemann had taught, and which was to become – thanks to the impulse of Felix Klein – the liveliest centre for research in mathematics. In Göttingen Volterra had the opportunity to see Schwarz, whom he had met in Switzerland. He then went for a few days to Berlin, where he met Leopold Kronecker, one of the most important German mathematicians of the day. This was the summer of 1891. Volterra returned to Pisa in September, but his academic life would soon undergo a change.

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[11]Vito Volterra, 'Henri Poincaré', Griffith Conrad Evans, trans. *The Rice Institute Pamphlet Volume Four* (Houston: Rice Institute, 1915), pp. 133–162; quotes from pp. 134, 138.

Chapter 3
The Cats of Torino

3.1 The Move

The 1890s opened with the important new phase of the transfer to Torino. We don't know exactly what the reasons were that led Volterra to leave the University of Pisa, where not only had he received his training, but where he had also been given a chair at a very young age. To be sure, the desire to 'liberate' himself from the watchful eyes of those who were by then his colleagues, but had just a few years earlier had been his teachers, must have played a role. It is easy to imagine that *Professor Volterra*, within the Faculty of Sciences, was still the *young Volterra* to the many older chair-holders who had known him when he was only 18 or 19 years old.

While we don't know the exact motives for the move to Torino, we can however, thanks to several letters, reconstruct the negotiations that made it possible. Everything happened in a short space of time. There were the first overtures, and Volterra did not deny his interest in talking, at least according to the letter of 11 July 1893 by Andrea Naccari, an experimental physicist who was first a professor and later the rector of the University of Torino: '...to Prof. D'Ovidio and myself, Ròiti had written some months ago that you were not adverse to coming here'.

Francesco Siacci, a professor of rational mechanics in Torino, known above all for his firing tables in use by the artillery, had decided to transfer to Naples in order to be closer to Rome, which was the centre of his activities and where he also served as a member of Parliament. It was at this point, to find someone to take his place, that the Faculty of Sciences unanimously 'called' Volterra. In a letter a few days later, on 15 July, Naccari commented with satisfaction on the turn of events: 'the way in which things have gone seems to me to be very suitable for you and for us. From the minutes of the Faculty, it doesn't appear that you have ever expressed a wish to come here, since you are perfectly free, and whatever your decision is, the fact does you honour. At the same time, however things turn out, it cannot be other than an honour to the Faculty that they appreciate your merits and seek you out wherever you are'.

Instead, for Volterra the most delicate phase had just begun. On one hand he had to lay the groundwork in Pisa to prepare for his departure. It was, after all, the place that had believed and invested in him. Thus he wrote to Dini, in July: 'Most esteemed Professor, I have come to look for you several times in recent days but I have never succeeded in finding you. I would like to tell you some news that I received the other day from Segre, that is, that the Faculty of Torino has called me there to succeed Siacci, who is going to Naples. This news reached me unexpectedly, and I have written to Torino saying that since it concerns a matter of importance I wanted to wait until I had had time to reflect on it'. On the other hand, he had to try to secure from Torino the best economic terms for the move. To be sure, this was not a matter of exorbitant amounts or bonuses. The place at stake was an appointment in higher mechanics – the Faculty of Sciences in Torino intended to assign that to him as well as the chair in rational mechanics – and a salary that was slightly above the usual amount. We are speaking of 2,200 Italian lire a year, the same salary that Siacci had earned. Today this would be equal to between 7,000 and 8,000 euros but, even for this amount Torino had to put pressure on the ministry because it such decisions were made in Rome. The day when Italian universities would be autonomous was still far away! It is here that Dini came in, with what Volterra did not hesitate to refer to as his 'adjustments'. In the attempt to change his former student's mind, and with many strings to pull in Rome – Dini had been a member of Parliament and was by then a senator of the Kingdom – he pushed the offices of the ministry to ensure that no particular increase of salary would be given to Volterra for the position, should he decide to transfer to Torino. Volterra then sought the support of Luigi Cremona, an illustrious mathematician who was the founder of the Italian School of geometry, also a senator, and future minister of public education in the Di Rudinì government of 1898. Volterra's letter was dated 13 October: 'I desire to engage you in the question of my transfer to Torino. Prof. D'Ovidio and the other members of that Faculty were so kind as to request of the ministry, to make the move easier, an increase of the honorarium relative to the position. The minister, from what D'Ovidio has told me, was disposed to concede that increase. It seemed to be by that time certain, when, following actions taken by the University of Pisa and especially by Prof. Dini, the promise was retracted. This is was Prof. D'Ovidio has announced to me today from Rome'. In the end, the long tug-of-war concluded with an agreement with which Volterra claimed to be satisfied. Thus, beginning with the academic year of 1893–1894, the young professor from Pisa moved to the University of Torino.

3.2   Torino and Peano

Among other things, the move meant Volterra's arrival in a big city. In the final years of the nineteenth century, the population in Pisa was about 30,000 inhabitants; that of the capital of Piedmont was about 300,000. It was not only a question of the number of people: in those years Torino could already boast of monuments and

public places as grandiose as Piazza Vittorio Veneto, the neo-classical Church of the Gran Madre di Dio, the Valentino park and castle, and the grand boulevards. Its major train station was Porta Nuova, which was now connected to France thanks to the opening of the Fréjus rail tunnel through the Alps in 1871. In addition to the university, since 1859 the Valentino Castle had been home to the Royal School for Engineers, which was the starting nucleus for what would become the Politecnico di Torino. In 1864 – the same year that the capital of the newly unified Italy was moved to Florence – construction began on the Mole Antonelliana, the striking building designed by architect Alessandro Antonelli destined to become the symbol of the city. Torino wasted no time reacting to the loss of its political primacy, transforming itself into Italy's major industrial centre. The focal point of the 1884 'General Exposition' in Torino was the pavilion dedicated to electricity. The production of electrical cables also began in the 1880s, with the company that would later become the Cavi Elettrici e Affini Torino, or CEAT, the first producer of electric cables and later a very large producer of car tyres, and the Savigliano workshops were founded for the production of railway materials. Above all, this was the eve of the founding of the Fabbrica Italiana Automobili Torino, better known by its acronym Fiat, in 1899.

The mathematical world was affected by the city's new industrial vocation, and by a certain attitude of pragmatism, more interested in the concreteness of the applications than in the depth of the theoretical investigations. Angelo Genocchi, trained as a lawyer but a self-taught mathematician about whom we will hear more shortly as the *maestro* of Peano, lamented the 'bad habit of throwing discredit on theoretical studies' for the sole purpose of highlighting the fact that in Torino 'the aim of mathematics teaching . . . is that of forming the majority of students to practise the art of engineering'. When Volterra arrived in the Piedmont capital at the beginning of the 1890s, the 'old guard' of Felice Chiò (1813–1871), Angelo Genocchi (1817–1889) and Francesco Faà di Bruno (1825–1888) had already passed away. The new names of greatest prestige were now those of Enrico D'Ovidio (1843–1933) and Corrado Segre (1863–1924) for geometry, and Giuseppe Peano (1858–1932) for analysis. To these should be added Francesco Siacci who, with his transfer to Naples, made it possible for Volterra to move to Torino.

D'Ovidio was not originally from Torino – he was born in Campobasso in the Molise region of southern Italy – and moved to the north only in 1872, when he won the chair in algebra and analytic geometry at the University of Torino, where later he would become rector. In 1905 he was named a senator of the Kingdom of Italy, although his appointment was accompanied by malicious rumours to the effect that the nomination was the result of a banal misunderstanding, the appointment actually being intended for his brother Francesco, a well-known philosopher who was also named a senator just a few months later. It was under the advisement of D'Ovidio that in 1883 Corrado Segre, then only 20 years old, earned his degree, leading Italian algebraic geometry to its full potential. His research turned out to be an particularly important link for inserting Italian studies in a broader international context. Segre used projective techniques to 'dialogue' with other investigations carried out, especially by the German school. He was able to complete, at the very

period when Volterra came to Torino, his study of the geometry of an algebraic *curve*, that is, the study of the properties of algebraic curves that turn out to be invariant under birational transformations. It was his school that would develop the study of algebraic surfaces that would become the greatest feather in the cap of the Italian tradition.

The rising star of analysis in Torino was Giuseppe Peano. After receiving his degree, he had begun to work in the university with Genocchi, whose lessons in differential and integral calculus he had edited for publication in 1884 under the title *Calcolo differenziale e principii di calcolo integrale*. The text effectively followed the lessons of the *maestro* Genocchi, but were revised by Peano to include 'significant additions and some modifications and various annotation'. It was precisely those 'significant additions', and the suspicion that his name had been used for an academic and editorial initiative of dubious scientific value that sent Genocchi into a rage, causing him to send a message to the mathematical journals with which he was in contact denying authorship, 'in order that nothing is attributed to me that is not mine, I must declare that I had no part whatsoever in the compilation of the volume cited'. His ire was then placated, partly because the volume that became known as Genocchi-Peano had in the meantime been received favourably with reviews and remarks that underlined its value and its pertinence. As the irony of fate would have it, Genocchi is above all remembered as the 'author' of Peano's text. For his part, Peano, after Genocchi's death, was named professor (acting) of infinitesimal calculus.

In 1890 the paths of Volterra and Peano crossed for the first time. Up to then they had had no occasions to meet or to exchange letters, except for a minor episode in November 1882. Volterra had complained that, in an article on the theory of integration, Peano had not mentioned his research and his paper 'Sui principii del calcolo integrale'. Peano responded amicably, and the 'incident' finished there: 'I am quite sorry about not having known earlier of your paper; I hope however to make honourable amends, if not before, in the publication of a textbook for calculus, set out according to the methods of Prof. Genocchi, of whom I was a student and am now an assistant. The first volume of this treatise is actually being printed now; the first printed pages are on sale here in Torino, for the convenience of students, copies of which I am sending you. I thank you most sincerely for the paper you sent me, and for the kind expressions in your letter. I am very pleased to have made your acquaintance, and please make use of me in any circumstance in which I can be helpful. I would be so thankful if you want to write to me sometimes, and if you have time to and wish to let me know your thoughts about the book I am publishing'. But we were talking about 1890. Genocchi had died on 7 March 1889, and the problem of substituting him (with Peano) in the coming academic year was immediately discussed. But for various reasons, it hadn't been possible to constitute a commission to decide. The competitive examination was postponed for a year, and this time Volterra was part of the commission (although he was younger than Peano by 2 years). In a letter of his to Betti, dated 10 October 1890, we find a brief summary of the outcome of the competition: 'I returned yesterday from Rome after having finished the work for the competition. Taking part in the commission were

Brioschi, Beltrami, Tonelli and Pincherle. Two equal votes of 48/50 were assigned, to Peano and Pascal. It is to be hoped that both come to enter into higher education'.

Peano was confirmed as a full professor in 1895. He had already made significant contributions to his field, some of which are still cited today in textbooks of analysis. He reformulated the definition of Riemann's definite integral by means of the concepts of upper and lower bounds of a set of real numbers, and above all, as regards its extension to functions of two variables, he defined the measure of a region of the plane. He gave a new formula for calculating the remainder term of a Taylor series. He is also known for his counterexamples – often 'ferocious' in their simplicity – with which he denounced overabundant hypotheses, errors and inexactitudes present in the most widely used textbooks. His existence theorem for the differential equation $y' = f(x, y)$, proven with the sole condition that function f be continuous, dates to the second half of the 1880s; it was followed by the exposition of a method of integration by successive approximations. The so-called 'Peano curve', which remains one of the most 'amazing' and counterintuitive results introduced into set theory by deductive rigour, dates to 1890. It played a fundamental role in the history of the concept of dimension: it is possible to find a curve, expressed by two continuous functions $x = f(t)$ and $y = g(t)$ that passes through all points of a unit square, while t varies in the interval [0,1], and thus it is not always possible to enclose an arc of continuous curve within an arbitrarily small area.

In the meantime, Peano also developed a keen interest in logic, not so much as an autonomous discipline as an instrument for rendering mathematical reasoning and its exposition more rigorous. The objective was that of decomposing and analysing the propositions of classical mathematics down to their smallest constitutive elements, in order to be sure that nothing more nor less than what is strictly necessary appears in them; the same propositions were then reassembled and rewritten by means of combinations of algebraic and logical symbols, or ideographs, leaving no room for misunderstanding and permitting them to be exactly and concisely formulated. In 1888 Peano published the *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann*, his work on geometric calculus according Hermann Grassmann's theory, which contained an introductory chapter on logic (as well as the first, original system of axioms for vector spaces). The following year he published his *Arithmetices principia, nova methodo exposita*, in Latin, containing the famous 'Peano axioms' that defined the natural numbers in terms of sets, which remain one of his most significant contributions.

In 1891 Peano edited the first issue of the mathematics journal *Rivista di Matematica*, which serves as a valuable source for documenting his progressive conversion to logic. He immediately put forth the idea of a '*Formulario*' aimed at collecting all the formulae and propositions of mathematics, written in a concise symbolic language that would facilitate their comparisons and cataloguing: 'it would also be a most useful thing to gather all the known propositions that refer to certain points in mathematics and publish these together. Limiting ourselves to those of arithmetic, I don't believe we will find any difficulty in expressing them in logical symbols; and then these, in addition to acquiring precision, will also acquire concision; and probably the propositions reflecting certain subjects of mathematics

can be contained in a number of pages no greater than that required by their bibliography'. The project for the *Formulario* would involve Peano intensely in the years that followed. The outcomes did not live up to what had been hoped for, but in the beginning the idea was well received. The final resolution of the 1894 congress of the French Association for the Advancement of the Sciences, for example, contained an item dedicated specifically to Peano's project: 'The great efforts made by Professor Peano and various of his colleagues for the dissemination of mathematical logic and the publication of a *Formulario matematico* are such that they contribute strongly to the desired end'.

As things progressed, there formed around the *Rivista* a group of young and combative young researchers who were primarily interested in logic and problems related to the foundations and history of mathematics: Giovanni Vailati (called 'the philosopher' by other members of the group), Rodolfo Bettazzi, Giulio Vivanti, Cesare Burali-Forti, Tommaso Boggio, Alessandro Padoa, Giovanni Vacca (a mathematics historian), Mario Pieri (a scholar of foundations, particularly of geometry), and others. The affairs of two of these collaborators also involved Volterra, in one case in the past, in the other the immediate future.

We have already mentioned Rodolfo Bettazzi (1861–1940) in the first chapter. He had come to know Volterra at the Scuola Normale, where he had been admitted as an day-pupil without a scholarship. Bettazzi received his degree in mathematics in June 1882, in the same session as Volterra. Then they remained less and less in contact, not least because Bettazzi was serving in the military far from Tuscany. We have, however, a letter from him dated 2 May 1883, congratulating Volterra on his brilliant performance in the competition in Pisa: 'I have learned here ... of your most deserved nomination to the position of mechanics and I congratulate you with all my heart. Forgive me if I have not done so earlier'.

While many things changed in the life of university students and of new graduates, other things stayed the same: Bettazzi too hoped for a possible career in the academic world, and so his friend, who had already gotten into the system, gave him an inside view of new developments and possible openings. A postcard from Volterra dated November 1883 informed him of the possibility of a scholarship for a year and Betti's intention to assign him a series of classes at the Scuola Normale. In effect, Bettazzi was given a position for a year to teach a course in number theory to second-year students at the Normale, but later events did not follow the path hoped for. The competitive examinations that he sat for ended up with an acknowledgment of his capabilities but without winning him a post. In the meantime, Bettazzi had written some interesting papers along the lines of Dini's work regarding the extension to functions of two variables of some fundamental results with regard to derivation and integration. He also published several articles of an educational nature and edited a monograph on the theory of magnitudes entitled *La teoria delle grandezze*, which aptly expressed his sensitivity towards the foundations of mathematics. He was rewarded by the Accademia dei Lincei with publication in the *Annali delle Università toscane*. Finally, he won the competitive examination to teach in secondary schools. In January 1891 he was called to teach in the Liceo Cavour in Torino, just a few years before Volterra would arrive in the city. As we said, Bettazzi would come to be

associated with Peano and his school, taking an active part in the project for the *Formulario*. Most importantly, in 1895 he founded the Mathesis Association for mathematics teachers, of which he was the first president.

Giovanni Vailati (1862–1909) came to know Volterra a few years later. Vailati, originally from Crema in Lombardy, earned his degree in engineering from the University of Torino in 1884; he then received a degree in mathematics in 1888 (the two degree programs had the same courses the first 2 years). Beginning in the 1892–1893 academic year, he was assistant to Peano for the course in infinitesimal analysis (in 1895–1896 he also assisted in projective geometry). This is how he described his first job in a letter written on 22 December 1892 to his cousin Orazio Premoli: 'In these past days my new teaching responsibilities leave me very little free time, not so much because of the number of hours at school (9 per week, of which 3 are reduced to merely sitting in on the lessons of the *boss*) as much as because of the preparation required and the working out of the arguments to be treated in order to avoid the risk of making some blunder'. In accordance with the lessons and instructions of the 'boss', Vailati wrote a set of lecture notes entitled *Applicazioni di calcolo infinitesimale* (Applications of Infinitesimal Calculus) comprising just over 400 pages, including numerous exercises. He also collaborated with Peano on the *Rivista di Matematica*, contributing some papers and brief reviews. His attention, initially focused on the study of algebraic structures, was increasingly drawn to problems regarding the foundations of algebra. In the meantime Volterra arrived in Torino, and in 1896 Vailati left Peano to assist the newly arrived professor in the course of rational mechanics. Still under Volterra's wing, for 3 years he taught an open course in the history of mechanics. The opening lectures of those courses, on the significance of the historic dimension of scientific research, would turn out to be among the most engaging of his reflections.[1] We will have more to say about Vailati in the pages to come, because he will be the one, at the end of the century, to suggest to Volterra the first bibliographical references on the topic of the relationships between economics and mathematics.

3.3   Integral Equations

First let's follow Volterra through his period in Torino, in the final 7 or 8 years of the century. He naturally continued to carry on with his research and produce a remarkable quantity of scientific works, although perhaps less intensely than during his years in Pisa. Some believe that this indicates that the young mathematician was slowing down, or perhaps a resting a bit on his laurels, now that he held a chair and was therefore under less pressure to further his career. This is far from the truth.

The list of publications for the years from 1893 to 1900 is still a lengthy one. The number of titles is considerable, in particular with regard to the study of polar

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[1] The texts of these opening lectures are contained in the second of the three volumes of *Giovanni Vailati Scritti*, edited by Mario Quaranta (Bologna: Forni editori, 1987).

wobble. This investigation was the focus of Volterra's attention for several months, and was significant both in terms of its contents and as an important step in the construction of his scientific personality. We will deal with it in the next section, as well as with the dispute that went along with it. This will give us an opportunity to see the strongly unified way in which theoretical investigations and applications are developed, as well as to get a sense of the growing authority that Volterra had earned within the mathematical community.

Another article dealing with a topic of physical–mathematical interest is 'Sul fenomeno delle 'seiches'' (On the phenomenon of the 'seiches'), which studies the variations of the water level measured in Lake Geneva. Like a tide of low amplitude and brief cycle time, the water rose an average of 40 cm. at the shores of the lake. The 'high tide' lasts for some ten minutes; then, with the same slow rate of speed, the water goes down to below the original level, to then rise again, lower again, and so forth. The beginning of the rhythmical motion of the seiches coincides with the disruption of the equilibrium of the air mass over the lake. It is a 'hurricane' that causes the first rise in the water level, followed by oscillations that very slowly decrease due to friction, but which can last as many as 4–5 days. The paper 'Sul fenomeno delle 'seiches'' was the text of the talk that Volterra gave at the 1898 congress of the Italian Physical Society, in which he proposed that the Society undertake an analogous study in Italy to shed light on the phenomenon's dependence on the shape of the shoreline, the depth of the lake, and other physical characteristics. For his part, Volterra added the analysis of the question from the point of view of theoretical hydrodynamics. Usually, the examination of the vibrations of a heavy liquid presumes some kind of regularity in the shape of the recipient, which is obviously not the case in the hydraulics of a lake. But this more general problem can be solved as well. The procedure followed by Volterra exploits the analogy between the phenomenon of the seiches and the vibrations of elastic membranes, where '... what is sought is a function that satisfies the equation $\Delta^2 \varphi +K^2 \varphi = 0$ and which becomes null at the boundary. The values of K, for which there exist non-null solutions, are the exceptional values from which are derived the vibration periods of the membrane'. This is an explicit reference to the most recent studies of Poincaré, which directly connected a problem regarding partial differential equations to the analysis of an integral equation.

The seiches thus lead to the theme of integral equations, which constitutes the most significant research carried out by Volterra during his years in Torino. The problem was to solve a functional equation in which the unknown function appears under the integral sign. Several problems of mathematical physics lead directly to this type of equation, but so did other questions first represented as differential equations, which it is then more convenient to convert into integral equations. For example, the problem of finding the function $y = y(x)$ which satisfies the differential equation $y' = f(x, y)$, with the initial condition $y(x_0) = y_0$, under a continuity hypothesis, is equivalent to seeking the solution of the integral equation $y(x) = y_0 + \int_{x_0}^{x} f(x, y(x))\, dx$. Already by Volterra's time, integral equations had a history of being used as instruments of analysis. They are found in the writings of Laplace and Fourier and above all, starting in the 1820s and 1830s, in those of Abel and Liouville. However,

they had always been studied in relation to a specific problem and to the individual integral equations to which that problem had led: '...there is no systematic means for effecting such inversions (the procedure for which is only known in particular cases) and there is not even a clue as to how to recognise in general when questions of this nature can be solved, and, when this occurs, if there is one solution only, or if there are more than one'.[2]

Volterra was the first to elaborate a general theory of integral equations, investigating the existence and uniqueness of the solutions and arriving at presenting a general method for their solution, that is, one which left the expression of the nucleus indeterminate.

The first works on this topic were published in 1896–1897. As was the case earlier with the terminology of functionals, Volterra did not yet use the expression 'integral equation', which had been recently introduced by the German mathematician Paul du Bois-Reymond, but had evidently not received enough notice at that point. Volterra preferred to speak of *inversions of definite integrals* to indicate the search for the functions $\varphi(x)$ that satisfy the (linear) integral equation $f(y) = \varphi(y) + \int_0^y \varphi(x)K(x,y)dx$ in which are known the functions f and kernel K defined for $x \in [0, y]$ and $y \in [0, a]$. This equation is still today usually referred to as a *Volterra equation of the second kind* to distinguish it from the *Volterra equation of the first kind* (when the unknown function φ appears only under the integral sign) and from the so-called *Fredholm integral equations of the first and second kind*, in which both of the limits of integration are constant. Formally, a Volterra equation can be seen as a special case of those of Fredholm, in which it turns out that $K(x, y) = 0$ for $y<x$.

The method used by Volterra to solve an equation of the second kind can be summed up in three fundamental principles: the *principle of convergence*, the *principle of reciprocity* (which for simplicity's sake we will omit), and the *principle of inversion*.

Principle of convergence: first assuming that the function $K(x, y) = -K_0(x, y)$ is finite and integrable, the *iterated kernels* are calculated:

$$K_1(x, y) = \int_x^y K_0(x, t)K_0(t, y)dt$$

$$K_2(x, y) = \int_x^y K_1(x, t)K_0(t, y)dt$$

$$K_3(x, y) = \int_x^y K_2(x, t)K_0(t, y)dt$$

It is thus proved that the series $\sum_{i=0}^{+\infty} K_i(x, y)$ is uniformly convergent.

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[2]V. Volterra, 'Sulla inversione degli integrali definiti', *Atti Accademia delle Scienze di Torino* **31** (1896): pp. 311–323, 400–408, 537–567 and 693–708.

Principle of inversion: given $s(x, y) = \sum\limits_{i=0}^{+\infty} K_i(x, y)$, the integral equation $f(y)$ $= \varphi(y) + \int_0^y \varphi(x)K(x, y)dx$ admits a single solution given by $\varphi(y) = f(y) + \int_0^y f(x)s(x, y)dx$.

Thus, thanks to Volterra the *theory* of integral equations was created, a theory which was soon enriched by the contributions of Erik Ivar Fredholm (1866–1927) – a Swedish mathematician who had studied under Mittag-Leffler – and Hilbert. Volterra's work also remains important from the point of view of methodology. The heuristic principle of the *passage from discrete to continuous* appeared for the first time in his paper 'Sull'inversione degli integrali definiti' (On the inversion of definite integrals), which we mentioned earlier. Volterra did not yet use this expression, nor did he emphasise it as he would in later works. In like fashion, in order to explain how he arrived at the solution to the problem, and to illustrate the 'naturalness' of the formulae found and the hypotheses introduced along the way, he presented integral equations which he had dealt with as 'limit cases' of an algebraic system of linear equations. It was algebra that suggested a route for the extension of analysis. It was the same logical structure that Volterra had erected in the passage from real analysis to functional analysis, when he needed to pave the way that would lead to the definition of a functional and then to its derivative: each level of formalisation, when correctly interpreted, contained within itself the guidelines for surpassing it. In short, continuity and discontinuity are both present in the development of mathematical thought.

3.4 The Dispute with Peano

Volterra wrote on more than one occasion that he had worked out the theory of integral equations before the years 1896–1897, when his first publications on the subject were published. A first mention and reflections on an integral equation of the first kind are effectively already present in his 1884 paper 'Sopra un problema di elettrostatica' (On a problem of electrostatics). His interest in the topic was, however, 'restrained' by a harsh dispute with Peano, which occupied him – intellectually and emotionally – for a good part of the 2 years 1895–1896 (Fig. 3.1).

We recounted earlier the first 'encounter' of Peano and Volterra. To be sure, Peano must not have been particularly enthused by the arrival of his young colleague in Torino. He judged this new event to be contrary to his own interests, because having another analyst in the faculty – and what a analyst! – even though he held the chair in rational mechanics, posed the possible risk of compromising his nomination to full professor. His initial tone, however, was that of the usual cordiality between colleagues. Peano invited Volterra several times to collaborate in the *Rivista di matematica*, and in response to this friendly pressure, Volterra finally, in 1893, proposed several papers on mathematical physics that were mainly didactic in nature to the journal for publication: 'the articles of which I am speaking would be four: (1) On potential theory; (2) On the mechanical interpretation of a formula of calculus of variations and Maxwell's theory compared to that of actions

Fig. 3.1 A young Giuseppe
Peano

at a distance; (3) On a problem of elasticity analogous to a problem of Dirichlet's
hydrodynamics; (4) On some formulae of the theory of elasticity analogous to a
formula of Gauss on potential theory'.

Only the first of these four articles was carried out and published. In fact, the
situation deteriorated suddenly at the beginning of 1895. To phrase it lightly, we
can say that at the origin of the dispute, on the part of Peano, there was a cat. On the
part of Volterra, there was a series of papers published in the winter and spring of
1895 in the *Atti* of Torino's Accademia della Scienze. The problem addressed was
the analysis of systems in which there are motions due to the actions of internal
forces. The specific reference is to the earth. Investigations carried out to establish if
and to what extent the continuity of the motion of the earth's rotation can be
modified by meteorological and geological occurrences (earthquakes, volcanic
eruptions, glaciations, etc.) were already characterised by a certain consistency
and tradition. Instead, in Volterra's opinion, what had remained relatively unexam-
ined where the cyclical motions that, even though present on the earth and within it,
do not noticeably modify the form of its surface or the distribution of the masses on
it. However, nothing excludes the possibility that such motions, which arise due to
the action of internal forces, can exercise an notable influence on the position of the
poles.[3] Volterra's works were aimed precisely at studying the motions of the earth's
pole caused by cyclic movements of the earth's surface, such as ocean currents,
atmospheric currents, the continuous motion of river waters to the sea, and so forth.

The first two papers, which formulated the problem and reported some initial
findings, were presented by Volterra at the beginning of February, on the occasion
of his election as a resident national member of the Accademia delle Scienze of

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[3]These movements of the poles 'do not have anything to do with the motions proper to the
terrestrial axis with respect to an inertial system of reference, which are the well-known motions
of precession and nutation. Instead, these are to be seen in a system of reference that is conjoined to
the earth, a system in which latitude is defined with reference to the equatorial plane perpendicular
to the earth's axis', G. Puppi, 'Vito Volterra e la fisica del suo tempo', *Atti del Convegno
internazionale in memoria di Vito Volterra* (Rome: Accademia dei Lincei, 1992).

Torino. During the elections, on a single ballot, the fifteen voters could indicate at most five names: Volterra was the first name listed on no fewer than thirteen ballots! The papers presented in February were followed by three more, presented again to the Accademia in the month of March, with the analytical solution of the general problem. Among the most favourable and most qualified reactions was that of the astronomer Giovanni Schiaparelli (1835–1910), who found the study to be of great theoretical interest. He observed, however, that in the specific case of the earth, 'when of the 3 moments A, B, C, one is notably greater than the other two . . . it is difficult to obtain movements of the pole of stable rotations with small stationary circulations of matter: perhaps it is possible to obtain small periodic oscillations that it would be highly interesting to examine'.[4] Writing directly to Volterra, he added, 'for the earth with small stationary motions there are only minimal periodic oscillations whose period is equal to that assumed for the stationary motions, combined with others of the usual Eulerian period. This is at least what I find upon a rough examination of the equations. It would be perhaps improbable to succeed in drawing out of this the Chandler period, 432 days'. Thanks to Schiaparelli's intervention, Volterra gained a greater awareness of the inevitable distance between the mathematical model and the actual phenomena. In any case, he immediately acted on the suggestion to study, in the case of the earth, the minimum periodic oscillations. In the sixth and seventh papers, read before the Accademia delle Scienze in Torino on 5 May and 23 June, 'wishing to apply the theory to effective terrestrial motions', Volterra simplified the analytical aspect with the suppression of some terms deemed to be negligible and assumed the hypothesis that the motions of the pole are already given as 'decomposable into a series of harmonic motions'. In the months that followed it would be instead the *analytical* or *functional* – as Volterra called them – aspects that became prevalent, with the study of the behaviour of bodies in which there exist internal motions that are cyclic, polycyclic or variable.

But by May and June, Peano had gotten into the act. His paper 'Il principio delle aree e la storia di un gatto' (The principle of areas and the story of a cat) appeared in January in the *Rivista di matematica*. The article referred to the discussion that had taken place at the Académie des sciences in Paris about why a cat, no matter how it falls, always lands on its feet. The question had arisen after a series of photographs had provided documentation of the dynamics of the fall. Peano's paper was an article of very high level popular science, and brilliantly written: 'this animal, left to its own devices, describes with its tail a circle in the plane perpendicular to the axis of its body. In consequence, by the principle of areas, the rest of its body must rotate in the direction opposite that of the motion of its tail; and when it has rotated by the desired amount, it stops its tail and with that arrests at the same time its rotational motion, saving in that way both itself and the principle of the areas'.

At that time no reference was made to the problem of the earth's rotation. Instead, Peano spoke with colleagues about the analogy between the phenomena

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[4] From a letter of 24 January 1895 from Giovanni Schiaparelli to astronomer Francesco Porro.

of the falling cat and the earth's motion, based on the fact that in both circumstances cyclic internal movements are capable of modifying the overall orientation of the body, but the chill with which his conjectures were met – and Volterra was certainly chilly – motivated him to address the question explicitly. This is what Peano wrote in a second communication to the Academia delle Scienze in Torino on 9 May: 'with respect to mechanics the question is identical. But to Prof. Volterra is due the credit for being the first to have proposed it. He made it the object of several papers presented to this Academy. . . The object of this note is the show how it is possible to calculate the displacements produced on the earth by the relative motion of its parts, and to apply them numerically. The calculation is made without squaring, solely by applying the principle of *areas*'.

Volterra's initially chilliness soon transformed into something much more severe in nature. He wrote to Valentino Cerruti at the beginning of May: 'Please pardon, esteemed professor, my insistence and bear with me. In these days my soul has been quite embittered by my colleague Peano. Without notifying me in any way, he has presented to the Academy a paper in which he repeated the same calculations and the same considerations I made regarding the motion of the pole, treating the thing almost as if it were new and making other statements that were likewise misleading, so that it was impossible to receive them benevolently. Moreover, he has attempted a numerical application beginning with the examination of the currents of the Gulf on the movement of the pole. I maintain that this last calculation was carried out in a way that was inexact, without taking into account that the data that he began with were not reliable and the results not at all in conformance with reality'.

This is the first document that testifies to the irritation with which Volterra began to follow the developments of events. The tones used in the private minutes of the Academy are still measured, not least because it is easy to imagine the care taken in choosing the expressions in order to avoid making the situation more tense than it already was. But the unpublished notes that Volterra used to prepare later interventions leave no doubt as to his state of mind and his harsh judgment: 'That Peano's note has no scientific purpose is made clearly evident from the following: that he merely makes a futile repetition of studies and research that I have already published, nor can a numerical application that he attempts and carries out with a strange precipitousness, basing himself upon unreliable data, justify his note. . .. Professor Peano states that the idea from which I took my cue is so simple that it occurs to everyone. I will not discuss such childish criticisms, but will only observe that the validity of an idea must be judged only from the importance of the applications, not from its greater or lesser simplicity . . . in his haste to denigrate the work of others Prof. Peano has shown a complete lack of preparation . . . The finding that the terrestrial pole is moving by a few meters a year towards Spitzberg is absurd! Anyone would be struck by such a ridiculous and absurd finding, but Prof. Peano, with the petulance of one who has no knowledge of a question, believes that he has solved it, when he has done nothing but misunderstand it. The absolute lack in Professor Peano of any knowledge regarding the problem [is shown by the fact that] he is ignorant of the most important works of mechanics by Euler, Lagrange, Jacobi, Hermite, etc.; or perhaps he believes that with a stroke of his pen he can cancel their

work? ... It is no pleasure to follow Prof. Peano in his lucubrations [and] along the path he follows in what I would call a beastly way.... It is certainly not by cultivating the field of so-called logic, for which both involvement and knowledge are superficial, that one prepares himself to study the problems of nature'.[5]

Volterra's criticism was so insistent that Peano was forced to withdraw a later communication presented to the Academy on 19 May. He had considered the special case found when the ellipsoid of inertia is a sphere; thus his calculations, applied to the earth, turned out to be unreliable. Peano sent notice of this by telegram. As Volterra told it: 'He recognised my error ... [freeing me] from the very unpleasant obligation of replying to him in print'. But Volterra couldn't avoid doing just that regarding a later note presented on 23 June, which in his opinion reproduced almost in its entirety the contents of the one that had been withdrawn. He thus wrote a condemnation of Peano's work that appeared in September in the *Rendiconti* of the Accademia dei Lincei. Peano's reply took the form of a note presented by the esteemed mathematician Eugenio Beltrami to the same Accademia dei Lincei. To make a long story short, the dispute that had begun in Torino had by this time moved to the most prestigious scientific academy in Italy, finally concluding with a letter from Volterra dated 1 January 1896 addressed to its president, Brioschi, in which he reiterated his own position, considering the question closed, and a final intervention on the part of Peano, dated 1 March, that repeated the findings of the previous note, without any mention, however, of the controversy.

The dispute, of which we have briefly sketched the most important phases, had thus grown up around a question of priority. Volterra accused Peano of not having informed him about his intentions to address a question regarding mathematical physics, in spite of their almost daily encounters in the faculty, and of having essentially copied his formulation, limiting himself to merely rephrasing Volterra's studies in the language of geometric calculus. Beneath a veil of apparent courtesy, 'Prof. Peano has really done a bad thing'. His numerical application, besides turning out to be clearly unreliable, maliciously insinuated that he had solved the question, while Volterra had, at best, merely posed it. Then, the dispute's development had broadened the controversy, and brought to light the gulf between Peano and Volterra, in terms of methodology and objectives. Peano had no real interest in the problem of the movement of the pole or in its solution; instead his main objective was to illustrate the full potential of a new language which, in his opinion, was able to make use of calculations that were much simpler, while at the same time making evident the unitary nature of problems that were apparently diverse. In reality, his faith in geometric calculus was so great that the project for its dissemination was carried forward with sometimes less than thorough rigour, and not always with in-depth regard for the specific nature of the questions treated. Peano liked provocations. He found controversy stimulating; in the subtle and brilliant logical analysis used to volley arguments back and forth, he identified a channel

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[5]The dispute is recounted in greater detail in Angelo Guerraggio, 'Le memorie di Volterra e Peano sul movimento dei poli', *Archive for History of Exact Sciences* 31 (1984), pp. 97–126.

through which scientific knowledge moved forward. Instead, Volterra favoured
another way of working, and was less interested in controversy. He was certainly
more concerned with the specific problems, their solutions, and the calculations
required to obtain them. Perhaps they might be less brilliant, but they were
definitely more useful. As far as content was concerned, Volterra could easily
show – and this was what occasioned the telegram – inconsistencies in Peano's
analysis. In general, his argument that it is not sufficient to understand the logical
structure of a problem is a convincing one: 'what is absolutely false is what Peano
says, that once the idea is posed, the question can be considered solved'. The
mathematician's work begins at this point, or at least, continues from it. The
calculations have to be done! This is where the long pages dedicated to the solution
of differential equations, to the use of elliptic functions, to problems of stability and
to the geometric description of the movements in question come in. Without the
ambition of 'inventing' a new mathematics, but with the effort to insert oneself into
a lofty tradition of mathematical physics –'after all, the procedure for integration is
that followed for every problem of rotation from Euler to our day, you can consult
any mathematics textbook'[6] – and the ulterior effort to remove hypotheses used for
convenience or introduce conditions that render the model still more realistic.

For Peano and his logic, the sentence was substantially without appeal. Volterra
was not the only one to pronounce it.[7] 'God liberate us from his symbols if these are
the results they lead to', he wrote Somigliana on 26 April 1896. The year before,
when the dispute was ongoing in all its asperity, Bianchi had written to Volterra on
21 May: 'such a great distance separates you and [Peano] in reality and in the
esteem of the scientific world, that the sole fact of Peano as a reviewer and critic of
your work, is already in itself supremely ridiculous. That he, with his mathematical
logic, wasn't capable of getting a spider out of a hole nor of finding the least
proposition of any value is notorious, and as for myself, for some time I have
believed that that is only a mask to hide the nonentity of his results'. Somigliana and
Bianchi were Volterra's friends, and had studied with him at the Scuola Normale.
These opinions, however, were not only held by them. They expressed a judgment
that was widely shared within the Italian mathematics community.

In scientific disputes it isn't always easy to draw a clean line between right from
wrong, even though what is being disputed belongs to the field of science, and even
when there is a sufficient amount of documentary evidence. Those involved in the

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[6]From a letter of 2 June 1895 from Volterra to Peano.

[7]The only 'dissenting' opinion was that of Beppo Levi (1875–1961), who had received his degree
in Torino and whose teachers had included Segre, D'Ovidio, Peano and Volterra. Levi worked in
real analysis, algebraic geometry, foundations of geometry, logic, and number theory. He taught in
the universities of Cagliari, Parma and Bologna. In 1932, writing about Peano's ideas and work in
the *Bollettino dell' Unione matematico italiana*, he arrived at maintaining what even Peano
himself had dared not claim: 'it was also Peano who provided one of the first and most notable
examples with a fine series of papers ... on the question of the displacement of the earth's pole,
which had become current in 1895–96 as a consequence of the so-called experiment of the cat, and
of a happy parallel noted by Volterra between that experiment and the astronomical phenomenon'

controversy inevitably tend to 'muddy the waters', cause the various planes of the discussion to overlap, and even attempt to change the rules of the game. The more properly scientific arguments become mixed up in personal rivalries or jealousies between specific schools. Nor are interventions in support of one or the other side on behalf of those involved always helpful. In the case of the dispute between Volterra and Peano, the verdict appears to be pretty clear. Volterra had no difficulty underlining Peano's lack of preparation, both theoretical (regarding mathematical physics), or at the level of applications to geophysics, at the very same time that Peano nourished an ambition to solve the question in a way that was innovative and radically simpler. Likewise, on the level of contents, Volterra easily showed his colleague's errors, as well as the pretentiousness of certain of his polemical insinuations.

The judgement is corroborated by successive developments. After the controversy with Peano, Volterra weighed in again regarding the movement of the poles with contributions that are still cited in the literature today, albeit for their historical value. In 1898, in his long paper entitled 'Sur la théorie des variations des latitudes' (On the theory of the variations of latitudes) he collected all of the investigations he had carried out in those years.[8] Instead, with the end of the nineteenth century Peano became increasingly isolated with respect to the most significant areas of mathematical research, placing all his bets on logical-linguistical developments, which would ultimately turn out to be less fruitful than expected.

The final years of the century: the international congresses

When all was said and done, Volterra came out rather well from the controversy. The 12–15 months we have just discussed were not among his most pleasant personally, but they gave him the necessary momentum to show his colleagues both in Torino and in Italy in general – we have already mentioned the important publications on integral equations, which followed shortly afterwards – that he was a 'serious' and trustworthy mathematician. His career moved forward. In 1894 he had been elected a member of the Società dei XL, and had also been made a member of the executive board of Palermo's Circolo Matematico. The following year, together with Corrado Segre, he won the national prize for mathematics awarded by the Accademia dei Lincei. In 1897 he took part in the foundation of the Italian Physical Society, and in that same year he became a member of the Astronomische Gesellschaft in Heidelberg.

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[8]Much later, in 1938, Volterra published his *Rotation des corps dans lesquels existent des mouvements internes* (full title: *Conférences sur quelques questions de mécanique et de physique mathématique: Rotation des corps dans lesquels existent des mouvements internes, Part 1*, vol. 4 of *Collection de physique mathématique* (Paris: Gauthier-Villars, 1938). At a distance of forty years from his first investigations of the subject, Volterra underlined their prevalently theoretical aspect: 'il ne saurait être question de tirer d'une telle étude l'explication du mouvements du pôle' (there can be no question of drawing from such a study an explanation of the movements of the pole). Volterra collaborated on this book with editor Pierre Costabel, who was later known primarily as a historian of science.

1897 was the year of the international congress of mathematicians in Zurich, which marks the beginning of the history of such international congresses. There had been a 'taste' of this kind of event in Chicago in 1893, organised by several American researchers with the decisive help of Felix Klein. Klein had taught at the University of Göttingen since 1886, but earlier, in 1872, he had been nominated to the University of Erlangen, where he had given the inaugural lecture that outlined what later became known as the Erlangen Program. The 1893 congress took place in concomitance with the World's Columbian Exposition, dedicated, as the name indicates, to Christopher Columbus and the discovery of America. Chicago saw the exposition as an opportunity to project an image of itself as more than just an industrial metropolis, and to show that culture in the United States was not the sole province of the New England states. The Expo was thus flanked by several scientific congresses. Volterra, though invited, couldn't make it to Chicago, as the timing of the congress coincided with his move to Torino, which was already complicated enough. On the other hand, not very many European participants went to Chicago. In addition to the congress of mathematicians, there was one for astronomers, organised by George E. Hale, just a few years younger than Volterra, of whom we will hear more later. Hale met Volterra in person in Rome in 1909, but already in 1893 he had many acquaintances among Italian astronomers.

After the Chicago congress, the idea of organising international meetings at regular intervals gained momentum. However, the problem that remained was who was to assume responsibility of the initiatives and what the most appropriate venue was for hosting the event. The schools of mathematics that were most representative and powerful were those of France and Germany, and in France the echoes of the battle of Sedan of 1870 and the Franco-Prussian war had not yet died down. It was Georg Cantor, one of the main inventors of set theory, who proposed holding the first congress in 1897 in a 'neutral' country. It was envisioned as a kind of second 'dress rehearsal', since the first official congress was scheduled to take place in 1900 in Paris. In the end, his idea passed, with Switzerland and Belgium proposed as possible venues. Finally it was pressure from the Germans in favour of 'nearby' Zurich that prevailed, even though the official story said that it was due to the Swiss tradition of hosting international events.

The congress took place in Zurich on 9-10-11 August, in the halls of the Federal Institute of Technology.[9] Just over two hundred mathematicians attended. Italy was well represented, both in the organising committee – geopolitics at the time tended to ensure the presence at each level of a German mathematician, a French, an Italian, and a member of a 'neutral' country – as well as in the audience. About twenty Italian mathematicians travelled to Zurich. Cremona was among the congress organisers. Invited to give the general lectures were Poincaré ('Sur les rapports de l'analyse pure et de la physique mathématique'), the Swiss-German

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[9]For a history of the first international congresses of mathematicians, see Angelo Guerraggio and Pietro Nastasi, *Roma 1908: il Congresso internazionale dei matematici* (Torino: Bollati-Boringhieri, 2008).

mathematician Adolf Hurwitz ('Über die Entwickelung der allgemeinen Theorie der analytischen Funktionen in neuerer Zeit'), Peano ('Logica matematica') and Klein ('Zur Frage des höheren mathematischen Unterrichtes). The proceedings of the congress did not, however, contain the complete text of Peano's talk, because of the singular form that he had chosen for his presentation. He maintained that 'the quickest way to get a clear idea of mathematical logic, of its scope, of its applications, and of its frontiers, is to see it in action', and thus he went on to read the principal propositions from 'his booklet *Logique mathématique*, which constitutes § 1 of volume II of the *Formulaire de Mathématiques*'. These were the years when Peano was beginning to reduce the classes he taught and even his public lectures to comments on his *Formulario*. In Zurich, his commented reading was in any case very useful: the actual text of the booklet was made up of fifteen pages inexorably full of formulae, one after another!

Volterra went to Zurich with his mother. He didn't give a talk, but he took advantage of the occasion to consolidate friendly relations above all with his French colleagues. He met Paul Painlevé and Emile Borel. The following year, in Paris, he met Henry Lebesgue. In 1898 he hosted the young René Baire (1874–1932) in Torino; Baire would later become known for the so-called *category theorem*, and several contributions to real analysis with the definition of semicontinuity. He had won a scholarship to the Faculty of Sciences in Paris, and in Torino he put the finishing touches on his 'thesis', discussing some parts of it directly with Volterra. The two of them had 'a meeting of the minds' on several questions regarding the continuity of a function with several variables. Baire dedicated his thesis 'Sur les fonctions de variables réelles', published in the Italian journal *Annali di Matematica Pura e Applicata*, to Dini and Volterra.

Volterra involvement in the international congresses began with the next one, and he eventually became, in the space of just a few decades, the most 'popular' mathematician, receiving no fewer than four invitations to give a general lecture.

In Zurich the appointment for the next meeting was confirmed for Paris in 1900: it was the groundswell of Cantor's proposal. The final lecture was then given by Émile Picard, president of the Société matématique de France: 'The general assembly of this morning has done the Mathematical Society of France the honour of choosing it to prepare the congress of 1900. Permit its president to thank you most warmly. I cannot, in this quality, make a better promise than to augur that the Mathematical Society will be able to do as well as the mathematicians of Zurich have done in a task that is always a delicate one. . . . Quite fortunately, we do not all have the same intellectual tendencies. Some, for example, the illuminators of science, prefer unexplored regions and lay the stakes for the future; others have a greater love for studies that can be taken to their extreme conclusion. The latter blast away the rocks, while the former then lay out the highroad. One loves to see the things in a geometric way, while the other prefers algebraic formulae. We also have the philosophers of mathematics, and this century that is just finishing has witnessed, as have all epochs, a great flirtation between mathematics and philosophy. This is for the best, on the condition, however, that this philosophy is very tolerant and does not suffocate the creative spirit. Let us guard against being exclusive and try to feel the same

Fig. 3.2 Émile Borel

Fig. 3.3 Paul Painlevé

sympathy for all conscientious workers. Let us also recall that in mathematics, as in women's grooming, fashion exerts a certain influence'.

The invitation to Volterra to take part in the 1900 congress as a plenary speaker was the definitive acknowledgment of his scientific authority. Poincaré

Fig. 3.4 Cousin Edoardo,
Vito's future father-in-law

wrote to him on 27 May 1899: 'My dear colleague, I have been charged by the
organising committee to ask you to consent to hold a lecture in one of the general
assemblies of the congress. All of us attribute great importance to your accep-
tance, and hope to receive a favourable response soon. You will let us know later
the topic you have chosen and the possible duration of your talk'. Volterra
willingly accepted the invitation, and formulated three different proposals. We
do not know the details, but in a later letter from Poincarè, we read, 'the three
arguments you proposed are all equally interesting; you may choose the one that
suits you best. Our preference, however, would perhaps be for Betti, Brioschi and
Casorati' (Figs. 3.2, 3.3, and 3.4).

Chapter 4
Rome, Public Appointments, the Politician

4.1 Matrimony and Paris

For mathematicians (and their historians) 1900 is the year of the International Congress of Paris and Hilbert's 23 problems. For Volterra, it was also the year of his marriage and his move to Rome. Actually, all these events – the marriage, Paris and Rome – took place in the space of just a few months.

On 18 February 1900 Eugenio Beltrami died. We came across him briefly in the previous chapter, in connection with the controversy with Peano (Peano was not yet a member of the Accademia dei Lincei and asked his illustrious colleague to present a note of his, again regarding the wobble of the poles). Beltrami figures large in the history of mathematics. He is remembered above all for his work in differential geometry, which showed the influence of Riemann's ideas and their dissemination during his Italian sojourn. This gave rise to his interest in non-Euclidean geometries and the creation of a first model of them on the pseudosphere.

Volterra began to act right away to try to move to Rome and take over the chair that had been held by Beltrami. In contrast to the first move, the reasons for this one were clear. First of all, Volterra wasn't completely happy in Torino. The controversy with Peano aside, he hadn't been successful in 'finding a niche' either in the city, or in its academic community (with the exception of a few good friends). Then of course there was the undoubted prestigious of the academic position, but there was also the allure of Rome. Beginning in 1870 the policies of the various governments – both left and right – had been alike in promoting Rome as the true capital of the united Italy. The city was not supposed to be only the seat of politics and administration, but of learning and culture as well. The transformation of the Accademia dei Lincei from an institution that was essentially local to a body representing national culture, and the inauguration in 1883 of its new home in the prestigious Palazzo Corsini were emblematic of the new direction taken. A new slogan was coined for Rome: *terza missione*, a 'third mission' to follow those of being first the centre of the ancient world and then that of the Christian world. Rome was to live its third exciting incarnation as the capital of cultural life in the new

Italy. Particular attention was paid to promoting the academic world and image of the university. Thus, for professors, the 'call' to Rome came to represent an acknowledgment of professional success and the crowning event of an entire career. A final reason for Volterra's interest in Rome were related to his family, and these were decisive, because they were related to those of his prospects of marriage. In Torino, Volterra and his mother lived on their own. Instead, Uncle Alfonso, his mother's brother, with whom Volterra had lived in Florence before moving to Pisa for university, lived in Rome. Cousin Edoardo, who had also done his share in making it possible for Volterra to attend university, had by then lived in the capital for many years. His business had been particularly successful and Edoardo had become a rich entrepreneur, owner of farmlands and buildings. Rome was therefore the place where the family could reunite. Volterra's mother, like many mothers, was also worried because her son – by then 40 – had not yet married. She had her eye on Virginia Almagià, the daughter of her cousin Edoardo. Of course, all of this went on unbeknownst to the future bride and groom. We can imagine the letters between parents and relatives as they wove the web, at first cautiously and with circumspection to test the waters, and then more and more openly. Finally even Volterra was involved in the project, and this then became another reason to move to Rome.

However, the outcome of his attempts to succeed Beltrami was far from certain. It appeared that Ulisse Dini was also aiming for the same position so that he could more easily reconcile his political duties with his academic ones, and Dini had an excellent reputation that seemed difficult to beat. Volterra's 'old' teacher was a politician with deep roots in the world of Roman administration, and he – an analyst – was sure to be able to overcome the difficulties of holding a chair in mathematical physics. In spring 1900, Volterra brought all of his heaviest artillery to the field. He had on his side his faithful supporter Ròiti, Pietro Blasnera in the Faculty of Sciences in Rome (professor of physics and founder of the Institute of Physics in Via Panisperna), and Stanislao Cannizzaro, professor of chemistry. Flanking these was the young Guido Castelnuovo (1865–1952), a mathematician and geometer who had been a student of Segre's in Torino, who had recently moved to Rome and was considered the most brilliant Italian algebraic geometer of his generation. These were important allies, but not enough to combat 'battleship Dini'. So Volterra asked cousin Edoardo to intervene. His future father-in-law contacted all his acquaintances, making the situation known to, among others, Ernesto Nathan, an important member of the Jewish community, son of an exchange broker, English by birth and accent, a follower of Mazzini, and a Mason since 1887. In 1888 Nathan had become an Italian citizen. He had also gone into politics, supporting Crispi and the historic left and by 1900 he was already an important figure in Roman politics; in 1908 he would even be elected mayor of Rome.

In the end, the solution seemed to lie in a request that the Ministry of Public Education add a position in the Faculty of Sciences. That would allow both Dini and Volterra to come to Rome. At the beginning of the summer, the situation was complicated by a crisis in the government that led to a change in the ministry, but commitments made earlier were maintained. Starting in the 1900–1901 academic

year Volterra was a professor in the Faculty of Sciences at the University of Rome, where he would teach for 31 years (and of which he would also be dean uninterruptedly from 1907 to 1919). His classes in mathematical physics and celestial mechanics began in November. Enrico D'Ovidio, his colleague in Torino had this to say in a farewell letter dated 12 October: 'I thank you for the news that you forwarded to me from Blaserna, although your transfer to Rome is truly hard for me; but more than anything I must wish you the best, without mentioning that your wife will be happy not to have to leave Rome. Thus I wish both of you, as well as your excellent mother, a happy and prosperous stay; my wife and all my family joins me in this'.

As fate would have it, after all this intense 'diplomatic' activity, Dini didn't go to Rome after all. He accepted the proposal to become director of the Scuola Normale, and remained in Pisa.[1] In the meantime, as we have already learned from D'Ovidio's letter, Volterra had married! The plans of his mother and cousin Edoardo had worked out for the best.[2] Vito had overcome his qualms about becoming part of such a wealthy family, ignoring the insinuations that he had contracted a marriage of convenience: the two young people liked each other. After a short engagement – only a month – they married on 11 July. René Baire, who had earlier had a scholarship under Volterra, wrote to congratulate him: 'Dear Sir, I have heard with the greatest pleasure the good news that you have told me, and I beg you to accept my most sincere felicitations for both your marriage and for your appointment to the Faculty of Rome'.

Photographs from that time show us a man of 40, not very different from the photos taken 10 years earlier, although by this time he appeared more mature. Instead, at the time of the marriage Virginia was a young girl of only 25. She would remain constantly by the side of her husband, and live until 1968. She would be a fundamental presence for her husband, her children, and her household. Children and grandchildren would come to know a woman who was affectionate but determined, who never stopped standing up for her own ideas even with 'such' a husband.

After the wedding, the bride and groom went away on a honeymoon in Switzerland, where they visited many of the places that Volterra had been on the occasion of the congress of 1897. They then left for Paris, in time for the opening of the second International Congress of Mathematicians on 6 August.

Paris was having an extraordinary summer. On 15 April the Exposition Universelle of 1900 was inaugurated, for which had been constructed the Gare de Lyon, the Gare d'Orsay, the Grand Palais and the Petit Palais. It remained open until the middle of November, and its vast spaces – which extended from the Place de la Concorde to the Champ de Mars – were seen by millions of visitors. On the occasion of the Expo were held the second Olympics of the new era, as well as

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[1] Dini would direct the Scuola Normale until his death in 1918.

[2] The family vicissitudes and events of this period are amply documented by Judith Goodstein in *The Volterra Chronicles*, op. cit.

cultural events such as the first international congress for electricity and the fourth congress international for psychology. Organised in close collaboration with the congress of mathematicians was that of philosophers, sponsored by the *Revue de métaphysique et de morale*.

Under Poincaré's direction, the mathematics meeting lasted an entire week, and was divided into six sections. With the exception of an inaugural session held in the Palais des Congrés, all of the meetings took place in the Sorbonne. On Friday 10 August, congress participants were received by the president of the French Republic. They had been expected to number about a thousand, but the Expo, with its crowds and high prices, had acted as a deterrent and attendance was lower than expected. Other factors were some errors in planning and a particularly hot summer. Thus at the congress there were just over 250 participants coming from 25 nations, including about 20 Italians. Guccia, Levi-Civita, Peano, Somigliana, Vailati and Veronese were all present, and Volterra figured as one of the eleven vice-presidents of the congress.

The keynote lectures were to be entrusted to a Frenchman, a German, an Italian and a member of a 'neutral' country. As we know, among the Italians it was Volterra who was chosen for this honour. He was joined by Poincaré, the German historian of mathematics Moritz Cantor, whose talk was entitled 'Sur l'Historiographie des Mathématiques' (On the historiography of mathematics), and Mittag-Leffler, who discussed the personality of Sophie Kowalevski, retracing 'Une page de la vie de Weierstrass' (A page from the life of Weierstrass). The lecture most looked forward to was undoubtedly that of Poincaré, the 'host' and one of the brightest stars in the mathematical firmament. He did not disappoint. His lecture, 'Du rôle de l'intuition et de la logique en mathématiques' (On the role of intuition and logic in mathematics), which was presented to a particularly attentive audience, was a condensation of his entire 'philosophy'.[3] For the French mathematician, rigorisation represented the mature phase of mathematics, but this alone was not sufficient: 'the science of proof is not the whole of science'. It is true that logical procedures play the role of decomposing every proof into a series of elementary operations, but understanding their correctness one by one does not mean having understood the sense of the proof. In order to appreciate the beauty of a building, 'it is not enough to ascertain the solidity of each part and admire the work of the mason, one must understand the plan of the architect' (p. 124) just as in chess, 'to understand the game, it does not suffice to know the rules for moving the pieces' (p. 125).

The Paris congress was the one that saw the liveliest debate on the adoption of a universal scientific language. It is the one that, in the time allotted for papers, included one of the first expositions of the theory of integral equations by the Swedish mathematics I. Fredholm. Above all, it was the congress during which David Hilbert (1862–1943), considered along with Poincaré to be one of the 'points of the diamond' of mathematical research, proposed those 23 problems that would

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[3] Henri Poincaré, 'Du rôle de l'intuition et de la logique en mathématiques', *Comptes Rendus II Congres International des Mathématiciens, Paris 1900*. Paris: Gauthier-Villars, pp. 115–130.

characterise the new century and keep mathematicians occupied in the decades that followed. In his *Mathematical Thought from Ancient to Modern Times*, Morris Kline wrote that Hilbert was the most important mathematician of the twentieth century. The vast range of his interests is reflected by the multiplicity of the disciplines in which he made truly significant contributions: invariant theory, number theory, foundations of geometry, the study of algebraic structures, Dirichlet's principle, the calculus of variations. In the history of twentieth-century mathematics, Hilbert wrote some of the most important pages in the field of integral equations and functional analysis with applications to geometry and physics that speak volumes about the unified nature of his vision of mathematics. He did not share Poincaré's taste for geometry; he was more interested in algebra. He did not share Klein's cultural vision, nor his capacity for organisation. Hilbert was instead 'the problem man', the mathematician who tested his skills against the newest and most complex questions, who thinks that mathematics essentially consists in solving problems. His method of grappling with them, solving them and describing the procedure used in their proofs ushered in a new style of mathematical writing. In a first moment, Hilbert had been invited to the Paris congress as a plenary speaker, but then due in part to absent-mindedness and in part to a bit of a delay in replying, the organising committee inserted his talk among those given in a specific session. The importance of his lecture was in any case immediately evident. The 23 problems he proposed involved various areas of mathematical research: questions of foundations (including physics), geometry, topology, algebra, number theory and analysis. The problems actually presented in Paris were only ten. The second one, on the axioms of arithmetic, immediately led Peano to intervene during the talk, to announce that the problem had in reality already been solved, and that the solution would be set forth by a member of his group. The reaction might appear excessive, but Hilbert had just touched on one of the central research topics of the group from Torino – with even mentioning them! – and Peano felt duty-bound to assert the Italian 'presence'.

But let's go on to Volterra, whose talk was entitled 'Betti, Brioschi, Casorati. Trois analystes italiens et trois manières d'envisager les questions d'analyse' (Betti, Brioschi, Casorati. Three analysts and three ways of viewing questions of analysis). He had prepared the text of the talk at the same time that he was preparing to get married, and put the finishing touches on it during his honeymoon, inevitably and understandably apprehensive about his first important talk given in front of an audience of such a high level.

The three Italian mathematicians he discussed had all died within recent years: Casorati in 1890, Betti in 1892, and Brioschi in 1897. They are traditionally credited with the birth of modern Italian mathematics, with the 'legendary' trip of 1858 to the primary capitals of European science (Berlin, Göttingen and Paris) to see how they were organised and consolidate relations with the most esteemed German and French colleagues and their institutions. This historiographical tradition was born precisely with Volterra's talk in Paris: 'In autumn 1858 three young Italian mathematicians set off together for a scientific journey with the aim of visiting foreign universities and come into contact with the most celebrated

scientists abroad in order to learn about their ideas, and at the same time, make their own scientific works known. This journey of Betti, Brioschi and Casorati marks a date worthy of being remembered: Italy was on the brink of constructing its own unity and participate in international scientific works by adding its own contribution … It is in great part due to their work, their teaching, the indefatigable zeal that drove the young Italian mathematicians to scientific research, to the influence of higher education, to the relationships they established between our country and foreign countries, that a modern school of researchers in analysis was born in Italy'. In the first chapter we briefly presented Betti, who had been one of Volterra's professors at the Scuola Normale. The research of Francesco Brioschi (1824–1897) ranged from algebra to analysis, from geometry to mechanics, and to mathematical physics. It was, however, to the field of algebra that he made his most lasting contributions, with innovative research in the theory of determinants and algebraic forms. After the unification of Italy, in Volterra's words, 'his activities immediately turned to politics. … While carrying out political duties in the Senate, he occupied himself with public works and engineering, thus giving proof of a rare, admirable kind of activity and of a spirit that was versatile and ever ready for new undertakings'. In 1863 – thanks to Brioschi himself – the *Politecnico di Milano* was founded, its aim to contribute to the formation of that new class of qualified technicians held to be indispensable for the nation's industrial development. One of Brioschi's students was Felice Casorati (1835–1890). A scholar of complex analysis, he would spread the ideas of Cauchy, Riemann and Weierstrass throughout Italy, thanks in part to the publication in 1868 of his monograph entitled *Teorica delle funzioni di variabili complessi* (*Theory of functions of complex variables*).

Volterra showed how the intellectual characteristics of the three Italian mathematicians complemented each other: Brioschi had a sensitivity and versatility akin those of a politician and entrepreneur; Betti 'only loved one thing deeply: scientific research that was disinterested and aimed at a high philosophical end'; Casorati 'lived almost exclusively for his students and for the school'. Their complementary natures could be discerned in their way of carrying out research as well.

When we speak of the past, we inevitable speak about ourselves and what we love. How then it is not possible to glimpse in the figures of the three mathematicians, thus depicted, three models that Volterra strove to keep in mind during the construction of his own scientific personality? Brioschi was a man of calculations, which was not off-putting to Volterra in the least: 'his mind, accustomed to unravel the inextricable things of real life, say through a dense network of formulae as through a limpid crystal'. Betti was a mathematician who instead preferred the systematic unity of a theory: 'it was more gratifying to him to think than to work in an automatic or mechanical way, and thus those long calculations that were the delight of the severe Lombard [Brioschi] were unbearable to him. … in his mind ideas became clearer and more evocative when they went beyond their strictly analytic significance to penetrate into the broader field of natural philosophy'. Finally, the works of Casorati 'almost all have that special imprint that reveals the author's aim of clearing up some obscure point or simplifying some result or treating a set of doctrines critically and methodically'.

For Volterra, these various characteristics were not fortuitous or strictly personal. Rather, they correspond to the different phases of development of that *theory of functions* that can be identified – another judgment often cited and by now part of shared historiographical evaluation – in many of the routes followed in nineteenth-century mathematics. 'First particular theories are formulated . . .. During this first phase there did not yet exist uniform methods and it was best to solve each question by devising, case by case, the methods that present themselves most spontaneously and were the most suitable. It was best, in the lack of general principles, to resort to long and painstaking calculations, but it was precisely from those that those principles then emerged, gradually, in all their limpid simplicity. . . . But this period of great discoveries, dominated by the desire to come quickly into possession of the truth that would be revealed following audacious intuitions, was followed by a phase in which philosophical concepts predominated, and a general method was sought that was capable of embracing the entire body of doctrines and encircling it within a singe frame . . . . In the last phase, the theories finally found their most important and most fertile applications, the most appropriate force for their dissemination, and the most suitable expression for their didactic exposition, after having been subject to a revision and discussion dominated by the most refined critical spirit that science has ever known'.

Volterra performed the role of ambassador and promoter of Italian science, proclaiming its merits and originality. He then concluded his talk with a warm tribute to the congress's host nation. The phrases he used were courteous and apt for the occasion, but the sentiments behind them were sincere. The reference to the solution of fifth-degree equations (and to the work of Betti, Brioschi and the French mathematician Charles Hermite, who was among those present) provided the opportunity for Volterra to underline the cordiality of the collaboration that had been begun: the trip of 1858 'marks the beginning of the cordial friendship between Italian scientists and the French scientist, a friendship that has lasted almost half a century, and was consolidated by the same sentiments of devotion to science and the same faith in the elevated destinies of mankind, the symbol of brotherly ties between the two nations of Latin Europe'.

4.2   The Inaugural Lecture in Rome

The return from Paris was the beginning of Volterra's life in Rome. His mother joined the young couple when they went to live in Via in Lucina, near Montecitorio, in an apartment in the historic Palazzo Fiano, which cousin Edoardo had bought in 1889. Vito, or better, Professor Volterra, began teaching classes at the new home of the university. That was the beginning of the 1900–1901 academic year. Just a few months later he received the invitation to give the official inaugural lecture traditionally read during the ceremony of the opening of the following academic year. This was undoubtedly an honour, especially for one who had so recently joined the Roman university. The occasion suggested that the speaker address a topic related

to his own research work but that he develop it in a way that was not overly technical and included frequent references to other disciplines. For a mathematician, this was a particularly daunting challenge.

Volterra oriented himself towards a discourse that would illustrate the new applications of mathematics to economy and biology. The choice was significant. Vailati, his former assistant in Torino who had been attended the Laboratory of Political Economics founded in Torino at the end of 1893 and was one of the first to put forward the proposal of teaching economics in secondary schools, was the intermediary who helped Volterra approach the most recent mathematical-economic problems, and in particular the work of Vilfredo Pareto. Volterra wrote to Vailati on 1 July 1901 in terms that were still quite general: 'For all that it is unimportant to me, I cannot get out of the very heavy responsibility of given the inaugural lecture for the next academic year at the University. What can I do that will not be uninteresting to scholars in other disciplines than mathematics? I have though about doing something *on attempts at applying mathematics to biological and sociological sciences*? What would you say about a topic like that? Is it alright or should I substitute it with something else? In that case, what books do you know about that might be helpful to me? You *who know everything* and who are so well-versed in the sociological sciences can give me excellent advise and I would be most grateful if you would write me something in this regard'.

The inaugural address, read on 4 November 1901, began right away with the theme of the kind of *curiosity* that characterises – or should characterise, as Volterra ironically put it – the attitude of scholars. The men of science to whom he refers are thus different from those that the French writer Anatole France described in the anecdote that we will give in a minute. 'Curiosity', towards both theoretical investigation and applications, is a constant presence in the scientific work of Volterra, the analyst and mathematical physicist. We have already mentioned it in relation to his invention of the concept of functional. Now, however, with the reference to economics and to biology, this constant is enriched with new contents. The motivations and applications that Volterra begins to think about go beyond the classic connection between mathematics and physics, which, as far as his training was concerned, and for the history of mathematical thought in general, was embodied by *calculus*.

Let's listen to Volterra's introduction: 'Anatole France, that acute and brilliant philosopher and novelist, the delight of many a sensitive reader, tells the following anecdote. Some years ago, he says, I visited in a great city of Europe the galleries of natural history, accompanied by one of the curators, who describe the animal fossils to me with the greatest of pleasure. He instructed me quite well up to the Pliocene terrane; but then, since we found ourselves faced with the earliest vestiges of man, he turned his head and to my questions he replied that that was not his display case. I felt my indiscretion. One mustn't ask a scientist about secrets of the universe that are not in his display case'. Volterra makes Anatole France's 'refined and some-times paradoxical spirit' his own, but he maintains that it would be a mistake to conclude that scientists are the least curious people in the world: 'let us instead consider that fact as a symbol that represents the natural and often justified

hesitance of those who dedicate themselves to study to expound ideas and pronouncements outside the field to which their thoughts are usually turned and in which they perform their scientific work'.

The mathematician possesses (or should possess) an even greater curiosity, because he possesses interpretative instruments that are particularly powerful. The paths by which he interacts with reality and problems that are not yet formalised are cleared by constructing those *mathematical models* that are given here one of their first descriptions: 'studying the laws according to which measurable entities vary, idealising them, stripping them of or attributing to them certain properties in an absolute way, establishing one or more elementary hypotheses that regulate their simultaneous or complex variations; this marks the moment in which one truly lays the foundations on which the entire analytical edifice will be constructed'. A model is a simplified representation of reality which makes it possible to introduce mathematical formalism and take advantage of its power. The simplification performed – underlining certain properties while removing others – depends on the subjective reading of the phenomenon in question and on the characteristics of the chosen formalism: 'moulding concepts in such a way that it is possible to introduce measure; then measuring; deducing then the laws; going from these to hypotheses; deducing from these, thanks to analysis, which, yes, is a science of ideal entities, but is also rigorously logical; comparing this to reality; rejecting or transforming the fundamental hypotheses utilised when contradictions arise between the results of calculation and the real world; and thus arriving at divining new facts and analogies, or being able, from the present state, to reason aboutwhat the past was and what the future will be; this, in the briefest possible terms, constitutes the birth and evolution of a science of a mathematical nature'.

The building of a model and its analysis tend to 'show only what it is useful to see ... and hide all that is superficial and would muddle the view', sometimes arriving at establishing a precise equality between phenomena that are apparently different. This is not only a question of economy of thought. Mathematical investigation can even arrive at unexpected conclusions, and 'no one can therefore tell the geometer to which wide open spaces the narrow and rocky path that his calculations make him follow will take him'. At the same time, the mathematisation of a science must not lead one to expect too much, or to believe in a new dawn. The construction of a model is a translation into a new language of facts and observations already collected in an area that is not yet formalised: 'between approximative reasoning, with which even those who don't know how to calculate can still in many cases predict the way certain phenomena will behave and the mechanisms of the forces that govern them, and the subtle reasoning of geometry, which, from a contrived set of algebraic symbols, in a manner that often causes even those most experienced in and accustomed to analytical disquisitions to marvel, leads to a result that specifies the behaviour of those same natural phenomena, there is not such a difference as everyone believed there would be'.

There was even greater reason in the fields of economics and biology, where the matter of applications was only at the beginning, to take an attitude of great interest in the potential of instruments and language of mathematics, but without being misled by unfounded hopes.

With regard to biology, Volterra cites Schiaparelli and his geometric models, making reference to 'questions of classification and evolution, questions that are in any case very closely connected, so much so that genetic theories tend to make one depend on the other' and to the use of methods of statistics, based on calculations of probability. The presumption of the English Francis Galton, that every natural phenomenon is quantifiable, at least in principle, had led to the founding of statistical psychology, with the definition of the concept of *correlation* and the application of the curve of normal distribution to the study of psychophysical characteristics of individuals. On the other hand, it was also in 1901 that Karl Pearson, a mathematician by training who was strongly stimulated by the works of Galton, founded the journal *Biometrika*.

With regard to the analysis of economics, the guiding thread of Volterra's inaugural lecture was provided by the analogy that had been drawn between marginalist theory and the theory of mechanics, which constitutes 'together with geometry, if not the most brilliant, certainly the most solid and secure of the fields of knowledge in which the human mind glories in'. Marginalism focuses its attention on the conditions that guarantee a situation of equilibrium to the economic system, from which follows a natural 'sympathy' for the path taken by mechanics. Thus, the *analogy* with this discipline constitutes the paradigm for development. Here a reductionist outlook is evident in Volterra's words. For economics as well, in order to access the methods of infinitesimal calculus the phenomena be heavily idealised. The economist must reduce the subject of his analysis to a *homo oeconomicus* and suppose that the quantity of goods available to him varies by continuous degrees. But the way the research develops justifies by a large margin the initial approximations that, as was the case in mechanics, are then reconsidered, leading to formulations that better respond to the nature of the phenomenon being investigated: 'We have already mentioned the vicissitudes undergone by the idea of force in mechanics; from the heights of metaphysics, it descended to the field of measurable entities. So too in economics it is no longer time to speak with Jevons of the mathematical expression of non-measurable quantities. Instead, Pareto begins directly from the idea of ophelimity, as he did in his *Cours d'économie politique* (*Course in Political Economy*), proposing to begin from purely quantitative concepts with his indifference curve, which coincides so nicely with the contour lines and equipotential surfaces of mechanics'.

The allusion to the evolution of the ideas of Pareto with regard to the function of utility is particularly interesting because it is not confined to the inaugural lecture, but reappears in another episode a few years later. Already in his first writings, Pareto had avoided any 'philosophical' disquisition on the origins and nature of pleasure and pain. The only premise was found in the notion of *ophelimity*, an 'entirely subjective' concept which expresses in quantifiable terms an individual's pleasure in consuming an assigned set of goods. In reality, the notion of ophelimity continued to have a dark side: what sense does it make to associate a certain number to the enjoyment of a given distribution of goods? Or to state that one pleasure is the double of another? It was precisely these critical observations that led to the change of view in Pareto that Volterra noticed in the passage from the 1896 *Cours*

d'Economie politique to the writing of some articles of a few years later, which were then made official in the *Manual of Political Economy*. In addition to not being rigorous, considering pleasure as quantifiable is superfluous. The fundamental equations of economics refer to simple facts of choice and can therefore be deduced independently from the notion of pleasure. This, in fact, disappears in the equilibrium equations, almost as though the calculation was charged with eliminating everything incorrect and inessential that the economist had put in.

In the 1906 article entitled 'L'economia matematica ed il nuovo manuale del prof. Pareto' (Mathematical economics and the new manual by Prof. Pareto), Volterra reviewed Pareto's *Manual* 'from the mathematical point of view, setting forth the impressions that a scholar of analysis receives while reading the treatise'. He was aware that the work lent itself to being studied and commented on from various angles, and for this reason he solicited various contributions. His own review was limited to the mathematical part, taking, however, a broad view of this, and not one restricted to the Appendix that contained the actual calculations: 'the concepts and processes of proof and logic form the essence of the mathematical method, and thus it can be said that this informs Pareto's work beginning with the third chapter, in which he introduces the general concept of economic equilibrium'. Naturally, particular attention is paid to the mathematical procedures that show themselves to be developed in greater depth with respect to the author's previous works. Volterra dwells on the concept of 'indifference line', reiterating and specifying – from the mathematical point of view – Pareto's thoughts. As long as exchange regards only two economic goods, it is effectively possible to ignore pleasure as an initial datum: there is no problem of integrability and it is always possible to go back to the determination of ophelimity. This is no longer true (except in particular conditions) in the case of three or more goods, and here the *Manual* is amiss in not discussing, or developing, ideas that Pareto had already worked with in a previous article: 'The passage from the case of only two goods to the case of three or more and the related discussion merit a more detailed examination than that contained in the manual. In fact, it is known that while a binomial differential expression $Xdx + Ydy$ always admits an infinity of integrating factors, an analogous expression that is trinomial and has a greater number of terms cannot admit them'. How apt Volterra's observation was can be deduced by Pareto's quick response, contained in an article entitled 'L'ofelimità nei cicli non chiusi' (Ophelimity in non-closed cycles) published in the *Giornale degli economisti* in 1906, which begins with this acknowledgment, 'In a brilliant manner the most honourable Prof. Vito Volterra observed ... I am entirely of this opinion'.[4]

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[4] On the relationships between economics and mathematics in Italy, see Massimo Galuzzi and Angelo Guerraggio, 'Volterra e l'Economia matematica', *Giornale degli economisti e Annali di Economia*, XXXIX (1980), pp. 11–12; Angelo Guerraggio, 'Economia e matematica in Italia tra Ottocento e Novecento', *Scientia*, 1986, pp. 13–39; Angelo Guerraggio, 'L'economia matematica in Italian tra le due guerre: Luigi Amoroso', *Quaderni di storia dell'economia politica*, VIII, 2/3 (1990), pp. 23–37.

Thanks to the examination by the French mathematician Emile Borel, the 1906 review of Pareto's *Manual* was also able to confirm the hypotheses contained in the 1901 inaugural address, according to which expectations of what mathematics can do should not be exaggeratedly high, but neither should they be exaggeratedly low. Volterra agreed with Benedetto Croce about the fact that the conclusions to which mathematical economics arrived could not be considered to be absolute, but this does not mean that the role of mathematics is nonessential and should be confined to a marginal place in the debates concerning economic thinking and its translation into the social context. The essence of the long quotation by Borel, taken from an article that appeared in the *Revue du mois* that same year, is that mathematics applied to economics can provide, for every problem, useful and original indications for arriving at an optimal choice. However, the objective sense of the conclusions to which the mathematisation of a science makes it possible to arrive should not be blown out of proportion. It is necessary to be cautious in the use of expressions – widespread even in everyday speech – such as 'mathematically true' or 'mathematically proven', since the hypotheses around which the mathematical reasoning develops can implicitly contain something more than a natural quantitative translation of any observations. 'There are nevertheless cases', wrote Volterra, 'in which calculation seems sufficient to determine our decision; but this depends on the fact that we have previously established a certain rule of behaviour'.

4.3 Duties, Honours and Research

The first decade of the 1900s was the period in which Volterra built up the majority of his fame and image, both in Italy and abroad. The 'call' to Rome and the lecture given at the international congress in Paris favoured the dissemination and appreciation of his works, and led to an 'incredible' number of offers and invitations, which Volterra didn't back away from. Just as 'incredible' was the work load he carried.

The years at the beginning of the century were crucial from the point of view of his family as well. In 1901 Virginia gave birth to a first son, who sadly died soon after. In 1902, his uncle Alfonso died (cousin Edoardo, who played such an important role in Vito's youth, and who was now also his father-in-law and with whom he continued to maintain a particularly affectionate relationship, would live until 1921). In that same year, 1902, his daughter Luisa was born. She would grow up to be a biologist and would marry Umberto D'Ancona, of whom we will hear more later in regard to Volterra's research on population dynamics. His second born, Edoardo, was born in 1904. He would be a constitutional judge in Republican Italy. The following year, 1905, was born Enrico, who after World War II would become a professor of aerospace engineering at the University of Texas. The family had grown large. The needs of the children, an economic wellbeing that was by then consolidated, and the wish to have a place away from the rush and tumble where it was possible to recover after stressful periods, led to the building of a house in the country, in Ariccia, not far from Rome. It was inaugurated in 1905. In 1906,

Gustavo was born, but he didn't live more than a few months. The fourth and last child, again baptised Gustavo, was born in 1909.

Some of these events (both happy and less happy) occurred when Volterra was far from home. We mentioned the numerous invitations and tokens of international recognition he received. Bearing in mind that travel was obviously by means of trains (or by boat, as we will see, in the case of America) and considering how long they lasted, Volterra's ability to make it everywhere is truly remarkable. We last saw him abroad with Virginia in Paris, on his honeymoon, in the summer of 1900. The next summer he was in England, where he visited London, Oxford and Cambridge. Here, welcomed by the astronomer George Darwin (second born son of the great naturalist), he was able to get to know the special system that governs university studies. In 1904, accompanied this time by Virginia, he met up with Darwin again – and with Kelvin and other celebrated colleagues – at the meeting of the British Association for the Advancement of Science, during which he was awarded an honorary degree from the University of Cambridge. Initially, Volterra had planned to visit one foreign country each year, but he was not always able to abide by this rule. After Paris in 1900 and England the following year, in 1902 it was the turn of Berlin. From Germany he arrived to Denmark, Sweden and Norway. Here, on the occasion of the one-hundredth anniversary of the birth of Abel, he was awarded an honorary degree from the University of Christiana. In 1904 – in addition to Cambridge – he travelled to Germany, to take part in, together with Castelnuovo and Guccia, the third international congress of mathematicians, held in Heidelberg. That same year he was elected a corresponding member of the Académie des Sciences de France.

Still in 1904, he received a government mandate[5] to oversee the reorganisation of the institutes for higher technical education in Torino, with the aim of founding a polytechnic institute and find 'a form [of training] that more aptly responds to modern conditions of study and to the pressing needs of industry'. The commission charged with drafting the initial study, of which Valentino Cerruti was also part, was presided over by Stanislao Cannizzaro, who assigned to him in particular the task of examining the models of the most renowned polytechnic schools abroad. Volterra set off once again. After a brief stay in Milan, on 13 February he was in Zurich, where he studied the organisation of the Swiss Federal Institute of Technology. He discovered the new trend of very specialised semester-long courses, in which the role assigned to practice was much greater than in Italy, taught by professors who personally negotiated their salaries with the administration. He then went to Germany, where he found similar innovations. In the German polytechnic institutes, teaching was quite technical and the role of theoretical mathematics was drastically reduced. The professors were called to teach without having to sit for competitive examinations, and were evaluated above all on their professional experience (rather than the number of their scientific publications).

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[5] To be precise, the royal decree is dated 17 December 1903.

Fig. 4.1 The country house
in Ariccia

Further – characteristic of Prussian polytechnic institutes – in order to be admitted,
students had to have had completed a year of apprenticeship in a work environment
in order to have developed a concrete idea of what organised work in a factory
consisted. Berlin, Leipzig and Göttingen were the sites of interesting meetings with
numerous colleagues, both mathematicians and non-mathematicians. In Göttingen
he was able to meet up again with Hilbert and, more importantly, with Klein,
whose managerial spirit impressed him (this in a mathematician who had given
certain proof of his scientific bravura!) (Fig. 4.1).

   He also appreciated Klein's search for points of contact with the sectors of
production that were most involved with technological innovation, sharing his
criticisms of those mathematical environments that were poisoned by conservatism
and contrary to relations with the outside world. The report that he would submit to
the Senate at the end of his investigation did not fail to underline the superiority of
the German model, which was worth imitating, while continuing in some way to
preserve the tradition of Italian education, which had produced 'excellent types
of scientific engineers, which are instead rare in Germany'. The areas in need of
improvement, in Volterra's opinion, were above all the experimental teaching of
mechanics, which were still lacking in Italy; mathematics teaching, which should
be reduced, though 'kept in capable hands'; and the duration of studies, 'since
young people will gain intellectually and morally by leaving student life behind
quickly and entering into practical life'.

   In 1905 he returned to Sweden. Thanks to a suggestion by Gösta Mittag-Leffler – the
close nature of their friendship is shown by the choice of the name Gustavo for

Fig. 4.2 Poincaré and
Mittag-Leffler, respectively
first and second from the left,
in a photograph from 1910

Volterra's youngest child – he was chosen by King Oscar II to teach a series of classes at the University of Stockholm. Volterra stayed in the Swedish capital for more than 2 months, giving a course in partial differential equations. His *Leçons sur l'intégration des équations différentielles aux dérivées partielles professées à Stockholm* (Lessons on the integration of partial differential equations given in Stockholm) were published a few months later. On his return to Italy he stopped in Paris, where, at Borel's house, he had the pleasure of meeting Marie and Pierre Curie, who had recently been awarded the Nobel Prize.

1905 was also the year in which Volterra was nominated a Senator by Giovanni Giolitti, Prime Minister of the Kingdom of Italy, an institutional acknowledgment of the political role that he by that time played. At this point his responsibilities increased further. In addition to his scientific duties, he now had political and institutional duties as well, even before the funding of the Società Italiana per il Progresso delle Scienze (Italian Society for the Progress of the Sciences, known by its acronym, SIPS) and the Rome congress of 1908. On top of all, Volterra was elected president of the Italian Physical Society for the biennium 1906–1908 (Fig. 4.2).

His family was worried by several ongoing symptoms of fatigue, and urged him to give up some of these duties, but Volterra remained firm. His mother was more or less on his side, and the most his wife and father-in-law were able to get him to concede was a longer holiday at Ariccia and, in a few months' time, his resignation from the duties of organising the Politecnico di Torino.

Amidst all this hubbub, Volterra still managed to carry out some research projects! Nor did he lack in motivation, although the rhythm was obviously not that of a few decades earlier. During the years when he was involved in moving to

Rome and those around the turn of the century, his list of strictly scientific publications was reduced to just a few titles, but in 1905 it began to increase again with his works on elasticity.

Apart from the paper on real analysis written in 1881 when he was still young, the years in Pisa were characterised by the creation of functional analysis. In similar fashion, the principal research projects in the period spent in Torino can be synthesised in the study of integral equations (to which, for other reasons as well, can be added the works on polar wobble). At this point, in the years leading up to World War I, Volterra was most involved with questions of mathematical physics: the theory of elasticity, distortions, hereditary phenomena. Going into these questions in depth would lead to an analogous development on the front of analysis, with integro-differential equations and the notion of composition of two functions.

As mentioned, the first writings on elasticity appeared in 1905. They were motivated by a paper by the German mathematician Julius Weingarten, which Volterra himself had presented to the Accademia dei Lincei: 'I am pleased to send you a reprint of a note that I have just published in *Rendiconti* of the Accademia dei Lincei on elasticity. I began with the considerations that you developed in your paper of 1901'. Weingarten had noted that there can exist equilibrium configurations for elastic bodies not subject to external forces – for example, an elastic ring from which a 'slice' is taken, a thin layer of a new material introduced in its place, and the edges of the cut resealed – in which nonzero internal stress can also be found. The example of the ring is that of a multiconnected body. The open question remained – Weingarten did not provide an answer to it – whether an experiment with simply-connected bodies would produce the same results, where a closed path could be continuously deformed until it was shrunk to a point. It was to this end that Volterra proved the theorem according to which 'if an elastic body occupies a finite space that is simply connected (acyclic) and subject to regular deformations, it will find itself in its natural state, when it is in equilibrium and is not subject to external forces', while, to the contrary, 'an elastic body in equilibrium that occupies a multiconnected space (cyclical) may not be in its natural state, that is to say, it may be in a state of tension, also when it is subject to external forces, its deformation being regular'.[6] The answer to the question left unsolved by Weingarten is thus negative: only 'a multiconnected elastic body, whose deformation is regular, can remain in equilibrium without the action of external forces. We obtain the equilibrium states by operations I called distortions'.[7] From the point of view of analysis, the difference in behaviour is due to the fact that in multiconnected bodies the displacements are not necessarily monotonic.

It was thus on the basis of observations of the effects produced by the distortions that it was necessary to modify the classic theory of elasticity with regard to the case of multiconnected bodies, since their behaviour is different from that of

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[6] The two quotations given here are taken from the paper 'Sur l'équilibre des corps élastiques multiplement connexes', *Annales de l'École Normale Supérieure*, 1907.

[7] Ibid.

simply-connected bodies.[8] The systematic study of what Volterra called *regular distortions* was taken up and generalised by Somigliana – at the suggestion of Volterra himself – and reappeared again in the classic treatise by the English mathematician A.E.H. Love, who, however, used the term 'dislocations'.[9]

Volterra continued to study elasticity, moving on, starting in 1909, to the study of hereditary forces. Hooke's Law postulates the proportional relation between stresses and strains, and does not take into consideration elastic bodies with a 'memory', even though these too are found in nature, in which deformations also depend on those preceding it. Such a body has a kind of memory of its past. The more elastic the body, the weaker its memory: 'Volterra, jokingly, used to say that stucco has a memory of steel, and remembers everything, while steel, especially when it is good quality, is very forgetful'.[10] This was the beginning of 'hereditary mechanics', which examines the cases in which every action leaves a legacy and the actual state of the system depends on all of its previous history: 'All engineers know, for example, that a bridge constructed a long time ago does not deform today under the action of a load like it deformed immediately after its construction. If the end of a horizontal elastic bar that is fixed at its other end is subjected to weights that first begin to increase and then gradually diminish, the body, when the load is being reduced, does not assume the same deformations it passed through when it was being loaded, and does not present the same bending in correspondence to the same bending weight. Therefore the actual deformation does not depend solely on the actual load, but on all the previous loads: for this reason it appears that it is possible to state the principle that every action that has had an effect leaves a memory in the body, which thus retains the memory of all the loads it has borne'.[11]

It is precisely because in a hereditary phenomenon the future state of the system does not depend only on the actual state (the so-called *initial conditions*) but on all the previous history, that differential equations are not the most suitable analytical tools for studying them. By analogous, inverse, reasons, 'to study in depth the problems of hereditary mathematical physics consideration of integral equations alone is not enough'.[12] What are needed are functions of lines, because if the state of the system is assigned by means of certain parameters, then its history is

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[8] In 1960, Volterra's son Enrico published the book *Sur les distorsions des corps élastiques (théorie et applications)* by Vito Volterra and Enrico Volterra (Paris: Gauthier-Villars).

[9] On Volterra's contributions to the theory of dislocations, see Franco Pastrone, 'Fisica matematica e meccanica razionale' in *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali*, Simonetta Di Sieno, Angelo Guerraggio and Pietro Nastasi, eds. (Milan: Marcos y Marcos, 1998). See also Franco Pastrone (with M. L. Tonon), 'Le origini della teoria delle dislocazioni: i contributi italiani', *Bollettino dell'Unione Matematica Italiana* (6), 1-A , (1982).

[10] The quotation from Gaetano Fichera is taken from Franco Pastrone, 'Fisica matematica e meccanica razionale', op. cit.

[11] Vito Volterra, 'L'applicazione del calcolo nei fenomeni di eredità', in *Saggi scientifici*, op. cit. (The text is substantially reproduced in chap. XIV of *Leçons sur les Fonctions de Lignes,* op. cit.)

[12] Vito Volterra, 'Sulle equazioni integro-differenziali della teoria dell'elasticità', *Rendiconti della R. Accademia dei Lincei* s. V, vol. XVIII (1909): pp. 295–301.

identified by those same parameters, considered, however, as functions of time. What are needed are the mixed type of equations that Volterra introduced, baptising them *integro-differential equations*. The differential part would regard, we might say, the future, while the integral part transmits what is inherited from the past. Volterra dedicated many scientific publications to these during this period, making use (for the solution of integral and integro-differential equations[13]) of the notion of *composition* of two functions, which he presented in a first paper of 1910.[14]

4.4   The Italian Society for the Progress of the Sciences

A letter from Stockholm, where Volterra was teaching the class on partial differential equations, to Giovan Battista Guccia (director of Palermo's Circolo Matematico) contains the first mention of a new objective that Volterra intended to pursue. This was at the beginning of 1905: '. . . probably this Easter, when there will be the congress for chemistry and the meeting of the Physical Society, various Italian societies, such as Zoology, Geography, etc., they will be attended by a delegate in order to lay the foundations for an Italian society for the advancement of the sciences, like the British one . . . The new Association, if it can be founded, would be autonomous but the various societies could serve as intermediaries between the new body and their own members'.

During the months that followed, the idea of establishing in Italy a society for the dissemination of scientific culture modelled after those already in existence in France, England and Germany, grew more concrete. The only precedent was the annual meetings of the Italian scientists held from 1839 to 1847, which were favoured by the climate of the Risorgimento and the spirit of unity that permeated the period. In the 1860s and 1870s there had been some timid and occasional attempts to revive them, but by that time the cultural and political context had completely changed, and the Palermo meeting of 1875 marked the end of the 'old' congresses of scientists.

Volterra officially presented his project for the founding of what would become the Italian Society for the Progress of the Sciences (known by its acronym SIPS) at the congress of natural scientists held in Milan on 15 September 1906, in part to celebrate the fiftieth anniversary of the founding of the Italian Society of Natural Sciences. In his talk, he discussed in some detail the earlier meetings of Italian scientists. He also mentioned the analogous associations active in other countries, dwelling in particular on the English one where he had been a guest – in Cambridge – in 1904 and which he had been able to observe firsthand. The new society would be

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[13] In 1913 he published his *Leçons sur les équations integralés et les équations intégro-differentielles*.

[14] 'Sulle funzioni permutabili', *Rendiconti della R. Accademia dei Lincei*, s. V, vol. XIX (1910), pp. 425–437.

able to respond to the need to overcome excessive specialisation only if constituted on new grounds, which were clearly different from those of earlier, academic societies. The risk that concerned Volterra was not so much that of giving rise to a consensus in which association and professional questions would take precedence over scientific concerns – by that time there already existed other forms of association in response to this social demand – but that of replicating past structures by now outdated that involved only a limited circle of members, one that might be characterised as 'scientifically aristocratic'. Instead, it was necessary to seek new channels of communication, break down old barriers and build new links among the various research groups, and between these and the social background that were both the natural catchment for consensus and the means for spreading their results: 'the new Association must have a broad base, it must be able to spread its roots freely throughout the country and embrace all of those who keenly love science, both those who have directly contributed to it, and those who only wish to master what others have discovered. In a word, the new Association must be scientifically democratic. It is true that some risk is run in following this concept, but it is worth running in order to do something youthful, vital and modern, as long as courage and good will are not lacking'.[15] The new society, aside from making it possible for individual scientists to meet, was intended to construct a network of all the agencies already working in the field of scientific knowledge and its dissemination. Volterra expected them to make an important contribution. Naturally, he guaranteed respect for their individuality and complete autonomy.

Volterra's proposal 'passed' in Milan. There were also other, similar projects set up by other scholars that converged with his own, and a commission was charged with preparing the launch of the new association. The constitutive congress held in Parma in September 1907 confirmed the prospect outlined in Milan. If anything, the memoranda circulated between the 1906 congress of natural scientists and that of Parma began to indicate a more precise political direction. The SIPS, in addition to providing an opportunity for democratic convergence in order to offset excessive specialisation and disseminate among researchers themselves a scientific culture that was less fragmented, was also charged with presenting the world of Italian science to the general public and to political powers. In Parma, during the opening ceremony of the congress, Volterra said, '. . .many harbour a desire for an imposing display of Italian science before the country, which perhaps does not yet fully appreciate the importance of scientific research, nor what a force it represents for the civil and economic prosperity of a nation'.

His entire discourse was farsighted, and included a lucid analysis of the relationship between science and society in Italy at the beginning of the twentieth century. The elements he identified as new were the growing awareness on the part of the public that economic and social progress derive from scientific progress, and the

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[15] The quotation is taken from the lecture given in Milan. Its text was published in the *Saggi scientifici* of 1920, reprinted anastatically in 1990 with an introduction by Raffaela Simili (Bologna: Zanichelli).

transmutations and contradictions that involved almost all disciplines. In spite of the term he used – 'crisis' –, these critical states were not seen as omens of a future bankruptcy of science. Rather, they documented a process of growth of a kind which Volterra had become almost used to.

Here is how Volterra described the first of these elements; even his inevitable recourse to the rhetoric of the day does not diminish their significance. 'The most modern discoveries, those which the greater part of our generation has witnessed, were seen by many (in contrast to what occurred more frequently in the past) to go from their birth and development in the scientific laboratories to spread among the workshops and invade the field of practical life. Thus the historic moment in which we are living presents us with the spectacle of the multitudes who, fascinated by those inventions which in a short time have become the source of much wellbeing and great wealth and influenced so profoundly social customs and consciousness, attempt to master scientific truth in its entirety, know it in its details, and what is most pressing, expect from science material and moral progress . . . . Thus has been established a continuous current that unites practical life with scientific life, and by natural correspondence and intimate virtue of things, those who make a profession of science have been drawn to the multitudes. Their existence is no longer closed within laboratories and studies; they feel driven to enter into intimate and daily contact with society and to participate in the life that animates the world. . . .Those modern, prodigious and enormous edifices, not smoking and appalling like the workshops of the past, but luminous and peaceful, where dynamos, the gigantic monuments of the present epoch, quickly and silently carry out their tasks, evoke, in their August, solemn and austere grandiosity, the monuments of another epoch: the ancient cathedrals that lifted their admirable spires to the heavens'.[16]

Allusions to a 'positivist' religion are evident in the reference to cathedrals, but the danger of an acritical faith with regard to science is avoided precisely by the difficulties in which scientific thought found itself at that moment. There was an evident and relentless need for specialisation, but at the same time there was a need to overcome it, because 'the different disciplines are so interwoven, that it is impossible to understand how one sector can progress beyond the present without knowing, and knowing profoundly, many others; and not only those held for some time to be similar, but also new ones that have now been shown to be strictly connected'. The difficulties that almost all scientific disciplines faced were pinpointed above all in the 'crisis of the philosophical thought that informs them'. It was the crisis of mechanistic principles. It was the 'war against the mythology of mechanics'. The proposal for the constitution of the new society was presented with great clarity and cohesiveness. The SIPS was created to respond to the needs produced by two elements which, as Volterra saw it, characterised that particular moment in science. On one hand, the 'inner crisis that is shaking up and transforming many doctrines makes a broad-based, open and direct discussion

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[16] The text of this lecture, 'Il momento scientifico presente e la nuova società italiana per il progresso delle scienze' is reproduced in *Saggi scientifici*, op. cit.

among researchers necessary, determining in them an urgency to make speak about the thoughts that occupy them, the doubts that torment them, the difficulties that hold them up, the hopes that drive them. Books and articles do not serve the purpose, nor can they ever be useful to this end; what is needed lies precisely in saying and understanding what one dares not publish or what will never be published'. The sacrosanct goal of overcoming of excessive specialisation would not be accomplished with a return to dilettantism, in which everyone discoursed on everything, but rather by opening up channels of communication among the various specialisations. The publicity given to the confrontation between scientists would also have a positive effect on the popularisation of science: 'Everything that the public is unable to understand from either books or lectures, will become clear when they can listen to and take part in the discussions between men of science, since it is spontaneous and lively dispute that reveal, in the most natural and truthful light, the germination and explication of those thoughts that are not usually disseminated by such apt means'. The 'internal' objective – greater awareness on the part of the scientific community of the perspective in which they operate and of the cultural relevance of their work – is accompanied by one that is more political: scientific progress is essential for civil progress. Political powers cannot fail to pay attention to men of science, who make up a fundamental segment of the nation's ruling class: 'but this is not all that the nation asks of the institution that is being founded; not only the satisfaction of the curiosity to know, but fruitful encouragement and spurring on of every fertile investigation and every new and vital research. Every day the men who are dedicated to industry, commerce, and practice of the professions have questions to ask of science, which is constantly being approached by a growing wave of people who are hopeful of receiving from it answers to the new, complex and pressing problems that face them, and believe it victorious over the new difficulties that are constantly arising'.

This marks an important juncture in the progressive enrichment of the contents of those *constants* in Volterra's scientific personality that we are attempting to make evident. As an analyst and mathematical physicist, he was naturally attracted to economic and biological applications. At this point, we participate in the 'discovery' of the economic and social value of scientific knowledge and the call to political powers to manage this relationship, a call that did not at all rule out the direct involvement of Volterra and other men of science. The path he had taken led to Volterra's being seen as an all-around intellectual of a scientific makeup. He is a figure who had attained considerable authority in his chosen professional field, but whose activities had by that time gone beyond the specific interests of his area. 'Since the beginning of modernity, the authority of the intellectual has rested on the claim to be acting and speaking on behalf of society as a whole. ... Being an intellectual implies social engagement. It is difficult to live for ideas and not attempt to influence society'.[17] At the beginning of the twentieth century, Italy had a great

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[17] Frank Furedi, *Where Have All the Intellectuals Gone?: Confronting 21st Century Philistinism* (2nd ed., Continuum Press, 2006), p. 34.

need of figures of this kind, although as the history of the century played out their importance tended to diminish. The other side of the coin is of course that there are only 24 h in a day, even for Volterra, who scrupulously planned his days – both the number of hours he would work and the number of pages he would write – even during the long trips abroad. His research was thus carried out with an intensity that would surely have been different had his professional duties been restricted only to studies and the creation of a particular school of mathematics.

Volterra made a different choice. Within the Italian scientific movement at the beginning of the century, he was the one who was most clearly aware of the political position at stake. The themes of scientific culture, the battle against particularism, a new synthesis centred on the achievements of science are not, however, exclusively his. One important adherent to the project for the SIPS was Federigo Enriques (1871–1946), another one of the 'greats' of the Italian school of algebraic geometry, who at that time was particularly involved on the philosophical front. In 1906 he published his book *Problemi della scienza* (*Problems of Science*). In 1907, as president of the Italian philosophical society, he organised its second convention in Parma to coincide with that of the SIPS, an ulterior attempt to bring the worlds of philosophy and science closer together. Then, elected to the scientific committee of the society promoted by Volterra, he gave Vailati confirmation of his support for the principals that inspired the SIPS: 'it seems to me that the Society of Sciences cannot thrive except on a democratic basis; and the banner of syntheticism, the battle against the artificial barriers of science, show that, in this case, "democracy" does certainly not denote a concept less elevated than science'.

There were of course those who did not believe in the project for the SIPS. This was the case of Guccia, to whom Volterra had turned in full faith, seeking help and collaboration. The founder of the Circolo Matematico of Palermo was not an easy person. He tended to use the interests of the Circolo – a truly admirable creation – as his only measure of judgement, and identify his own professional success with its development. He also tended to view other projects with suspicion, believing them to be plots, or in any case manoeuvres hostile to his 'baby'. In its objects the project for the SIPS appeared to him to compete with his hopes for the Palermo Circolo. He initially responded to Volterra's invitations with a measure of scepticism, advancing the hypothesis – not improbably – that the fledgling SIPS would run aground: 'I will tell you frankly, on the basis of that bit of experience acquired in Italy, that I have little faith in the success, in our country, of an association similar to those in England and France. . . . In what other country have you come across the phenomenon of secondary school (and perhaps university) professors who meet in the name of Science, swearing allegiance to the government of the king in order to receive raises in pay? We all say that this reprehensible, and that's fine. But why expose ourselves to seeing the Association for Progresses in the Sciences deviate immediately from the purpose for which we want to found it?'. A later letter dated 28 June 1906 is even more explicit: what emerges is almost a private businessman's logic applied to the field of scientific culture. Guccia was interested in the results of the actions taken, rather than the method used. He preferred to be marooned in the solitary, oligarchic leadership of the Circolo rather than see it become part of

a broader reality whose drawn-out internal dialectic would provoke the serious risk of paralysis. The following long quote documents the dispute with regard to politics, which a significant part of Italian society and culture at the beginning of the century identified with an ongoing, bureaucratic negotiation that was incapable of avoiding a whole series of questionable vetoes: 'I'll tell you right away that the idea of founding an ITALIAN ASSOCIATION FOR THE PROGRESS OF THE SCIENCES is an excellent one. But it is necessary to find the "MAN", the "Energy", capable of leading such a difficult undertaking to completion. If you have this man, this energy *in pectore*, if you have truly found him in Italy and are willing to tell me who it is, in that case I might be able to say that the "idea" is liable to being carried out successfully. But if this man doesn't exist, allow me to tell you, with my customary frankness (and availing myself of that experience that I have acquired in these things) that you may be setting yourself up for disappointment! The "deliberations" of "Congresses", of "Academies", of "Commissions" of every kind and nature serve, and have always served, to given a legal guise (I would call it that) to work already done, or that is about to be done, BY A SINGLE INDIVID-UAL, by an energy that imposes itself! But one shouldn't expect more than this, in the practical arena, of such "deliberations". They remain "vows". If the zeal of some willing souls want to make more of them, the "vows", after some time, turn into "failed attempts"! ... Therefore, in my opinion, the praiseworthy attempt to form an Italian Association for the progress of science is destined to failure, or at least to meet the same fate as the one that died in 1875 with the Palermo congress, in which the most eminent men of Italian participated, and during which were pronounced stupendous discourses, rich in promises for the future of the Association, but which, unfortunately, contained ... politics, the microbe that in Italy enters everywhere and ... kills everything, especially science! Politics! Here is the great enemy of science in Italy! Here is why institutions that prosper and flourish in other countries never take hold in Italy! Try to inaugurate a congress or a library without an oration by His Eminence the Minister and related patriotic speeches, all of which basically say the same thing, which is this: that without Victor Emmanuel, Mazzini and Garibaldi there would have been, for example, no wireless telegraph! Meanwhile, in these speeches, the progress, future, and needs of science are relegated to last place, as an "in conclusion"! Now say, in this state of confusion of various things and ideas, if in an environment like this one, it will ever be possible for an institution that must have as its ONLY aim progress and the development of science, to arise and develop? In my opinion, no! Unless, I repeat, there emerges a man of great authority and phenomenal energy, who, free from any ties whatso-ever to politics, is willing to and can resolutely tackle the arduous problem, which is separating science from politics in Italy!'

In spite of the dismal prospects outlined by Guccia, with the congress of 1906 the SIPS was launched. It was immediately placed under the high patronage of the king, and the organising committee included senators such as Giovan Battista Pirelli, scientists such as Orso Mario Corbino, and men of high standing such as the banker Bonaldo Stringher, general director of the Bank of Italy from 1900 to 1930. There was also a certain degree of participation by women, seeing as how the by-laws

made it possible for women to join the society – an innovative aspect that is far from negligible. Sustained by the backing of colleagues such as Blasnera and Cannizzaro, Volterra was elected the first president. He remained in that office until 1909, when he was succeeded by the chemist Giacomo Ciamician.

The first secretary of the SIPS was Alfonso Sella, son of the textile magnate and statesman from Biella, Quintino Sella (1827–1884), to whom Volterra was particularly tied by both ideals and politics. Quintino Sella was one of the leading figures of the historic right: he had served in the government more than one, holding positions of absolute authority, and he is remembered above all for his policies for balancing the budget. In reality, he is a key figure for shedding light on those circles which, in certain times in Italian history, have toyed with the idea of a different kind of development for Italy, one based on innovation and technology. Trained as a scientist, later a professor of mineralogy at the school of engineering, Quintino Sella was also president of the renewed Accademia dei Lincei, and during the 1870s he had been the decisive campaigner for the reorganisation of the University of Rome (to which he had called, among others, Cannizzaro and Blaserna). It was Sella who coined the motto 'third mission' for Rome, which, after first having been the centre of the ancient world and then that of the Catholic Church, had arrived at a 'third season' of flowering as the capital of the Kingdom and thus as a fulcrum of cultural life in the new Italy. There are, then, obvious reasons why Volterra liked him; he was also the inspiration for Brioschi.[18] Among the members of the ruling class following Italian unification, Quintino Sella was the statesman who – by dint of both background and sensitivity – best understood the role played by science in Italy's economic and social progress.

4.5 The Rome Congress of 1908

During the same months that he was carrying out the project for the SIPS, Volterra also found himself involved in the fourth international congress of mathematicians. We can imagine that discussions in the family circle about this time were particularly, shall we say, vivacious. His wife, his father-in-law (in part) and his mother insisted that he slacken the intensity of his activities, using all the medical knowledge they possessed to convince him that this was necessary. In the end, they managed to make him promise to resign from his involvement with the Politecnico di Torino, but in the meantime he had founded the SIPS and had assumed responsibility for the organisation of the international congress of mathematicians!

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[18] See *I Lincei nell'Italia unita*, Giovanni Paoloni and Raffaella Simili, eds. (Rome: Giorgio Bretschneider, 2004), and with particular regard to Brioschi and the political and scientific environments of the period, see *Francesco Brioschi e il suo tempo* (*1824–1897*), Saggi I, Carlo G. Lacaita and Andrea Silvestri, eds. (Milan: Franco Angeli, 2000).

We find a first glimpse of the ideas Volterra was developing in a letter to Guccia dated 16 July 1904 written by Corrado Segre, his friend and former colleague from his days in Torino: 'Prof. Volterra wants the 1908 congress of mathematicians to take place in Italy, and more precisely, in Rome. He spoke to me about this in early June during a meeting of the members of the Lincei (the mathematics section). But everyone told him that he had to come to an understanding with the Circolo Matematico, or better, with you, in order to have your advice and help'. Thus we see that Volterra had immediately involved in his project both the Accademia dei Lincei and the Circolo Matematico of Palermo, which was represented the only association for mathematics then in existence in Italy, and which in the course of just a few years, thanks to its journal, the *Rendiconti*, had achieved a noteworthy international standing. In this case as well, Guccia's response was initially circumspect. With time, however, it changed into one of full agreement, although inevitably the role of the Circolo in the congress and its organisation would be overshadowed by that played by the Accademia dei Lincei, which was located in Rome. He could not in any case not accept Volterra's proposal: it represented the best possible opportunity to realise his dream of an international prize that would carry his name, which Guccia hoped to institute in concomitance with the congresses.

The Italian proposal was presented in Heidelberg in 1904, at the end of the third international congress. Volterra was there (accompanied by Virginia), along with Guccia and Castelnuovo, with whom Volterra intended to begin work right away during the organisational and planning phases of the congress. The idea of hosting the 1908 congress in Rome passed rather easily – the 'diplomatic' negotiations of the preceding months had been fairly intense and paid off well – to the disappointment only of the English delegation, which had presented a similar proposal of its own. At stake was not only the venue of the next congress, but third place in the ranking of world mathematics, after France and Germany, which had hosted the 1900 congress in Paris and that of 1904 in Heidelberg, on the heels of the 'trial run' in Zurich in 1897. With the acceptance of the proposal, Italy's occupying the third place was recognised officially and unanimously. It was the culmination of a long 'rush' that had lasted the whole 50 years since the nation's political unification. Volterra thanked those present 'for the honour you have done us in choosing Rome as the venue of the next congress. I propose holding the congress in spring of 1908, leaving it to the committee to set the exact date'.

The organising machine was soon set in motion, in summer 1905. The first steps were to find the funding to print the informative material and the proceedings, and to set up a social program that was worthy of the image Rome to be transmitted abroad. It was also necessary to form a committee to delineate the scientific part of the congress and make the more 'political' decisions, including the choice of the plenary speakers and those responsible for the various sections. Finally, it was necessary to frame a sufficient picture of the questions that earlier meetings had left unresolved and which merited being taken up again.

The thorniest question was that of the presidency of the congress. Volterra was not yet 50 years old, and was considered too young to assume directorship of an

international assembly. That kind of title would have explicitly sanctioned his leadership of the Italian mathematical world, giving rise to no small degree of professional jealousy. The better choice would be of 'grand old man' like Dini, but the mathematician from Pisa immediately let his intentions be known: he would have nothing to do with the idea! In the end, an agreement was reached on an 'institutional' name, that of Pietro Blaserna, president of the Accademia dei Lincei and vice-president of the Italian senate. Blaserna had been one of the major figures behind Volterra's 'call' to Rome, and with him as president of the congress as well as the organising committee, Volterra could in fact act unhindered. The fact that Blaserna was a physicist and not a mathematician was swept under the rug.

Other problems soon found a solution as well. The promised funding arrived – public and private, with particular support from insurance companies – and a date was confirmed, somewhat counter-current, with a decision made to hold the congress in April rather that the 'classic' summer period. The team composed of Castelnuovo, Guccia and Volterra immediately arrived at a good understanding. Guccia renounced any scientific responsibility in order to concentrate on important organisational aspects – promotion of the event, preparation of congress materials, publication of the proceedings, and so forth – not only making a noteworthy contribution, but also offering concrete proof of the Circolo Matematico's willingness to collaborate. Volterra settled the most difficult questions, assuming responsibility for contacts with the political sphere, the Italian administration, and many of the mathematical circles abroad. The actual work of organisation fell to Castelnuovo, who was growing increasingly worried that Volterra was spreading himself too thin, accepting commitment after commitment, many of which took him abroad.

The congress opened on the morning of 6 April, and promised well. There were more than 500 mathematicians present, with almost 200 people more accompanying them. It was a record attendance with respect to the earlier congresses in Zurich, Paris and Heidelberg, as Volterra underlined in an interview printed in the *Corriere della Sera* that same Monday. The opening ceremony took place in the great *Sala degli Orazi e Curiazi* in the Campidoglio. The king arrived at 10:00, wearing the uniform of a general and escorted by cuirassiers, and was received by the prefect of Rome, the mayor and by several members of the organising committee. Then the official discourses began. Ernesto Nathan, who had been elected mayor, reiterated the mission of this 'third Rome' and passed the word to the government representative. It was at this point – after the minister for public education had finished his speech – that Volterra took the floor. His was the final ceremonial discourse, and at the same time, the first of the talks that had a specifically mathematical content. The king stayed to hear his whole talk, doing him the honour of leaving the hall only after Volterra had finished his inaugural lecture.

Following the inauguration on Monday morning, the congress wound along through the week following a routine that was by that time well established: the mornings consisted of parallel sessions for specific themes, while the afternoons were reserved for the plenary lectures. This was rounded out by the social program, which was particularly appreciated by congress participants and those

accompanying them. There were also some new features. The general lectures were eight rather than four as before. Noteworthy among these was that by Poincaré – read by Gaston Darboux because Poincarè had fallen ill while in Rome – whose title, 'L'avenir des mathématiques' (The Future of Mathematics) made no secret of its wanting to reiterate the 'task' that Hilbert had assigned to the mathematicians of the new century in Paris. Another new feature was introduced into the sections – this as well promoted by Volterra – of a subsection dedicated to economic and financial applications, with specific reference to actuarial mathematics. Then there was the award of the 'Medaglia Guccia' – the international prize so ardently promoted by the patron of the Circolo Matematico – to Francesco Severi, the young rising star of algebraic geometry. There was also the launch of the International Commission on Mathematical Instruction (ICMI), charged with the task of preparing for the next congress the material that would document the current state and future trends of mathematics teaching in the secondary schools of the various countries.

As we said, Volterra's speech was both the last inaugural speech and the first lecture of the congress. His aim as 'host' was that of illustrating for his guests, both to open the congress and as a sign of hospitality, the progress that had been made in Italian mathematics. His journey through time began with Galileo and his quotation that the great book of philosophy is written in the mathematical language, noting that this became even more momentous in modern times, when 'the characteristics proper to Italian genius' are integrated with 'new methods of teaching and learning, and the fruitful emulation that arose from the clash of opposing tendencies', taking into consideration as well the legacy of experience drawn from the mathematics of other countries.

Volterra recalled the principal achievements attained by mathematical physics and its first two 'champions', Betti and Beltrami. For geometric studies, he recalled Cremona, who 'predominates and stands out'. Like a good host, he was benevolent and impartial, neglecting no one, not even Peano, with whom he had been involved in the bitter dispute of a decade earlier. If anything, he intentionally forgot to mention himself and the results for which he was internationally known. With regard to analysis, he of course cited Dini. With a combined regard for analysis and geometry, he couldn't refrain from mentioning the Italian vice of introducing too-rigid classifications, noting that it was necessary instead to emphasise the fact that research in analysis and that in geometry had much in common. They differ not so much in content as in the procedure used; in any case, when going from analysis to geometry or vice versa, we are not 'crossing the line between two encampments, each of which is armed against the other'.

Volterra's history of Italian mathematics in the second half of the 1900s was necessarily condensed into a few brushstrokes. Some of his judgements, however, would remain and would come to define new lines of enquiry. The aspect that emerges most clearly is the political one. His reconstruction of the most recent events in Italian mathematics demonstrated the importance of the Italian Risorgimento not only for politics, but for culture and science as well. It was no coincidence that his first words carried his listeners back to 1860, the year in which the

dream of Italian unification became reality, and in which Cremona inaugurated the teaching of higher geometry at the University of Bologna. It was the non-coincidental concomitance between political progress and scientific progress that Volterra wished to underline. Before, the scientific systems of the individual states were practically non-existent. Before, it might have happened that when two chairs were assigned, a algebraist and a scholar of ecclesiastic law might have been mistaken for each other, and the mathematician was named to the chair of law while the jurist found himself teaching algebra. Before, the number of university chairs was absolutely insufficient. Then came unity: 'joyful hopes illuminated those souls in which satisfaction with the Nation, so recently and with such fatigue acquired, was united with an aspiration to the highest scientific ideals'. No one should then wonder if 'in following the development of the sciences a sudden transformation is observed'. For Volterra, who would unfortunately be contradicted too many times in the course of the twentieth century, the country had gained the awareness that 'a nation that wants to be free and great does not live by soldiers and railways alone, and that a united Italy would be ill-fated if it did not go back to occupying the position in arts, letters and sciences that had distinguished it in other times'. This was the same logic that would lead Rome to believe in its 'third mission': there is 'one cosmopolitan object we cannot not have in Rome: that of science'.

Volterra's speech was addressed to the foreign guests, but certainly also to those in power in his own country. All of the route traced by him, up to the recent founding of the SIPS, led to his being considered a *leader* of the scientific community and its representative before the political powers. His image was that of a man of science well grounded in the traditions of the Risorgimento and capable of indicating precisely the road Italy had to take if it wished to intensify its policy of modernisation. In the presentation of Volterra's *Opere matematiche*, Castelnuovo spoke explicitly about the 'leading role in the Italian world of science' that Volterra assumed at the beginning of the century.

The Rome congress concluded with a festive air, with a visit to Tivoli and an *arrivederci* until the congress of 1912 in England, in Cambridge, which had lost its bid to host the third congress to Italy. The success of Volterra, the organising committee, and the whole Italian mathematics community was evident. Poincaré wrote: 'For some 30 years now, the Italian mathematics movement has been quite intense, both in Rome and in universities of other provinces: I would have to cite a large number of names that enjoy an honourable place in the history of science, but seeing them gathered in this congress best renders the idea of what has gone on in Italian scientific life'.[19]

The glow dimmed a bit only after the congress, due to the (consensual) break-up of the collaboration between Guccia and the organisers in Rome. The founder of the

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[19] The article by Poincaré, who had to follow the progress of the congress from his hotel room after falling ill in Rome, appeared in the French newspaper *Les Temps* as a chronicle of the scientific meeting.

Circolo Matematico had already expressed some doubts in Heidelberg, when the Italian organiser of the congress had to be named. Already at that time he had had a hard time swallowing the central role inevitably attributed to the Accademia dei Lincei. During the congress itself, the fuse was lit by Blaserna's inaugural lecture, who gave credit exclusively to the Lincei. Blaserna's speech was of the highest official nature, given in the presence of the king and all the other authorities. This led to the decision, immediately communicated to Blaserna, that the Circolo intended to tear up the agreement signed earlier and would no longer publish the congress proceedings. Thus the collaboration that had begun well and had been carried out in the best possible way, reached a bitter end. Guccia complained in particular about the behaviour of the three members of the board of directors of the Circolo, who also figured in the organising committee for the congress and who, in his opinion, hadn't lifted a finger to defend the role of the Circolo. The three members were Castelnuovo, Cerruti and Volterra. None of them were reconfirmed as board members for the following term. The exclusion of Volterra naturally caused the greatest outcry, given the repeated proofs of amity that he had shown to Guccia. But Volterra held no grudge over the exclusion, and would be one of the few mathematicians of note who participated, in 1914, just before Guccia's death, in the celebration honouring the 30-year anniversary of the founding of the Circolo Matematico.

Chapter 5
Days of Firing from a Dirigible

5.1 The Thalassographic Committee and Travels to the United States

While it's true that the years at the beginning of the century were particularly lively and rich in initiatives for Italian culture in general, and specifically for the scientific movement, Volterra's energy was singular and genuinely limitless even for that period. The international congress in Rome had hardly finished, with the unpleasant aftermath that we mentioned, before he immediately threw himself into the planning for what would become the Italian Thalassographic Committee.

We will have a chance to discuss marine biology and oceans when we arrive to the post-war period, as well as the studies on population dynamics, but we have already had a glimpse of Volterra's interest in problems regarding nature and their mathematical treatment – in this case, fresh water – in the paper on seiches and the proposal made to the Italian Physical Society, which had not failed to accept his invitation to undertake a study in Italy similar to that of the seiches carried out at Lake Geneva (see Chap. 3). At the congress of natural scientists in Milan in 1906 (which we mentioned in connection with the founding of the SIPS), there had been not a few speakers who had taken up the gauntlet, resulting in the installation by the Military Geographical Institute and the Physical Society of two limnological stations at Lake Garda.[1] Moreover, during his visit to Sweden in 1905, Volterra had been impressed by the oceanographic studies carried out by the countries of northern Europe and their usefulness when applied to problems regarding fishing.

The problem in natural science that increasingly attracted his attention during these years was precisely that of how to transfer experiments such as those performed in Sweden to the Mediterranean Sea, in order to study hydrographics and biology systematically. Some steps had already been taken in Italy. There had

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[1]More detailed information can be found in S. Linguerri, *Vito Volterra e il comitato talassografico italiano* (Florence: Olschki, 2005).

been oceanographic research to study the currents, and there had been zoological explorations to prove the existence of ocean fauna in the Mediterranean. Some hydrographic companies had undertaken the collection and analysis of biological materials. In Volterra's view of the state of the art of the field, he believed it was then necessary to give new impetus to these initiatives, assuring them greater continuity. This also meant transferring scientific knowledge to the fishing industry, taking it out of the sphere of merely empirical activities and formulating a rational utilization of marine resources.

The Thalassographic Committee was created as a subcommission of the SIPS at the end of 1909, with a resolution by the president and the board of administrators. This was during the months when Volterra, having completed his 3-year term as president from the founding of the SIPS in 1907–1909, was succeeded by Ciamician. More generally, the founding of the Thalassographic Committee is indicative of the vitality of the areas of hydrography and limnology at the beginning of the century, favoured by several important factors (such as the Zoology Station in Naples and the Naval Hydrographic Institute in Genoa) and by greater awareness on the part of state authorities of the economic usefulness of having more exact knowledge of the physical and biological conditions of the sea. As we noted, the process leading up to its founding had begun at the congress of natural scientists in Milan in 1906; it was there, while he was promoting the foundation of the SIPS, that Volterra listened to all the talks calling for increased attention to problems of the oceans and began to cultivate the idea of an entity that would support such studies and their practical applications. He worked on the project in the months that followed, until finally, in spring 1909, as president of the SIPS, he called a meeting to give a formal start to the constitution of the committee. By this time Volterra moved easily through the byways of the ministries of state, and thus right way the ministers of Agriculture, Industry and Commerce and Public Education were invited to take part in the temporary working group, which would soon become the organising group for the foundation of the Thalassographic Committee. The honorary presidency was offered to the minister of the Navy.

The effective presidency was entrusted to Volterra, as president of the SIPS. He didn't wait long to make his influence felt. The Thalassographic Committee did not even officially exist yet, and already a debate had grown up about some questions that would turn out to be crucial for the future entity: the creation of a large, autonomous network for oceanographic research projects, the establishment of laboratories managed directly by the Committee, the establishment of a publishing venue, and funding. Above all, an operating plan was immediately developed for scientific ocean voyages. Thus, in the month of August, a torpedo boat owned by the Italian navy left port for the first of four annual ocean study voyages proposed by the (not yet constituted!) Committee. It is significant – thinking of Volterra's future studies on population dynamics – that the Adriatic was decided upon as the location of choice, motivated by the fact that Venice and Trieste were home to important administrative and scientific institutions, and by the existence of commercial treaties with Austria that gave priority to the problem of fishing in the Adriatic.

So, Volterra led the entire process of setting up the Thalassographic Committee. Then, at the end of 1909, he left the presidency because his 3-year term was up. After a formal resolution for the constitution of the new entity, the first plenary meeting of the Italian Thalassographic Committee took place in Rome in January 1910. Its mission to provide incentives for research in theoretical and applied biology was confirmed, along with its taking the necessary steps to see that the results were transmitted to the fishing industry. Soon, however, the private nature of the Thalassographic Committee (which, for the moment, was a 'simple' offshoot of the SIPS) showed itself to be clearly inadequate for reasons both institutional and financial. Thus the idea of detaching the committee from the SIPS, transforming it into a national body funded by the state, made headway. Once again, Volterra's involvement was not lacking: thanks to his involvement, the parliamentary red tape was gotten through particularly quickly, and by mid-July 1910, Victor Emmanuel III had signed the decree that officially established the Royal Italian Thalassographic Committee. Volterra was unanimously elected vice-president of the 'new' committee,[2] which, in collaboration with the Ministry of War, underlined the affinity between meteorology and thalassography, adopting the slogan *by air and by sea*, and even establishing a aerological service for gathering data useful for air navigation.

It would be wrong, however, to think that the 'domestic' commitments of the SIPS and the Thalassographic Committee would be enough to keep Volterra completely away from his research and from its reverberations abroad. In the 1909–1910 academic year, he gave a series of lectures in Rome on integral and integro-differential equations, which were collected and published (in French) with the title *Leçons sur les équations intégrales et les équations integro-différentielles*. Also in 1909 he accepted an invitation from Clark University in Massachusetts, which had been founded by millionaire Jonas Gilman Clark. To celebrate the 20-year anniversary of its founding, the university asked Volterra to give a series of lectures on topics in mathematical physics, and at the same time awarded him an honorary degree.[3] He then took advantage of being in North America to visit other cities in the United States and Canada. This was his first trip to North America. The following year he would visit South America: after a brief stop in Spain and visit to the University of Barcelona, he flew to Buenos Aires as an official representative of the Italian Ministry for Public Education to take part in the celebrations for the 100 years of Argentine independence. He also spoke at the Inter-American Scientific Congress, giving two talks on relativity and integral equations. Afterwards he undertook a long excursion that took him to the south of Argentina, Patagonia and Brazil. In 1911 he (only) went to Geneva, again for a series of lectures regarding mathematical physics (Fig. 5.1).

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[2]An accurate reconstruction of the affairs of the Thalassographic Committee and its activities is found in Linguerri, *Vito Volterra e il comitato talassografico italiano*, op. cit.

[3]For more information abut Clark University and more in general about trends in superior education in America in the second half of the nineteenth century, see J. W. Dauben and K. Hunger Parshall, 'L'evoluzione della ricerca universitaria: Johns Hopkins, Chicago e Berkeley', in *La matematica*, vol. I, C. Bartocci and P. Odifreddi, eds. (Torino: Einaudi, 2007).

Fig. 5.1 Volterra sent this postcard (20 August 1909) from New York to his children to shown them how high American buildings were

Next he was in Bologna. Although it was held in Italy, his participation in the fourth international congress of philosophers, organised by Enriques, could not pass unnoticed. Although he had always claimed to be a simple amateur in questions of history and philosophy, Volterra was invited to preside over the section of logic and theory of science. For twentieth-century Italian culture, the Bologna congress remains a significant event, because it was during the congress that the scientific movement – and Enriques in particular – broke definitively with the neo-Idealism of Croce and Gentile. In an interview given to the *Giornale d'Italia* at the end of the congress, Croce bluntly dismissed Enriques as a dilettante who undertook 'the effort of [organising] congresses of philosophers that are as praiseworthy as my own efforts would be praiseworthy and disinterested if I were to organise congresses of mathematicians. I have much esteem for Enriques both because I know him to be a good mathematician and because for him philosophy is, if not a possession, at least a need. It is only that since he cannot fulfil this need in substance, he fills it with words'.

Returning to Volterra's international travels, in 1912, he went to London for the one-hundredth anniversary of the Royal Society, of which he had been nominated a

foreign member 2 years earlier. This had been preceded by his election in 1908 to the Imperial Academy of Sciences in St. Petersburg and the Swedish Academy of Sciences, and was followed by his election to the American National Academy of Sciences in 1911, thanks to the proposal by George Ellery Hale. He also went to Paris in 1912, invited by the Sorbonne to present a series of lectures on functions of lines.[4] There he met Edgar Odell Lovett, director of the newly-founded Rice Institute in Houston, who asked him to return to the United States for the school's inauguration. For his part, Volterra told Lovett about the particular gifts of Griffith C. Evans, a young American student who had a scholarship to study under Volterra, and who had been received by him with great cordiality. Evans was invited to several Sunday lunches with the Volterra family in Rome and in Ariccia, 'interrogated' repeatedly about the United States, its economy, and its scientific policies, and he would remain in contact with Virginia Volterra until the early 1960s. During the first post-war period Evans became an esteemed analyst and mathematical economist; later transferring to Berkeley, he contributed greatly to its reputation as an important centre for research and teaching. His career began, however, at Rice Institute, thanks to Volterra's recommendation.

Volterra accepted Lovett's invitation to visit Houston in 1912; it was his second trip to the United States. In addition to Texas, his tour included lectures at the University of Illinois, Columbia University in New York, Princeton and Harvard. He was particularly impressed by the vitality of North American society, and by an economic model that was able, thanks to a direct relationship between business executives and those working in institutions of higher education, to carry out what in Germany had only been possible thanks to the intervention of the state. He saw the close cooperation between businesses and educational institutions as forming the basis of the colossal processes of growth that was already in evidence in American society. The creation of new universities and centres for scientific research was the answer to the demand for education that arose from the development of an industry strongly based on technological innovation. The words he pronounced were not only suggested by formal courtesy: '... permit me to express the feeling of admiration that I feel in visiting this new and grand country: an

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[4]The text of the lessons, revised with the collaboration of the young French mathematician Joseph Pérès, was published under the title *Leçons sur les fonctions des lignes,* op. cit. This also contains a first, more explicit formulation of the *passage from discrete to continuous*: 'tout à fait semblable à celui par lequel on passa de la somme à l'intégrale et par lequel on arrive aux opérations plus générales d'intégration dont nous avons parlé. Est-il possible de se borner dans la philosophie naturelle aux fonctions d'un nombre fini de variables? ... On ne fait ainsi qu'un examen approximatif de phénomène, mais on entrevoit facilement qu'il y aura des cas, où, pour approfondir d'une manière convenable la question, il sera nécessaire de passer du nombre fini au nombre infini d'éléments variables' (quite similar to that by which one passes from the sum to the integral and by which one arrives at the more general operations of integration that we have spoken about. Is it possible to limit natural philosophy to functions of a finite number of variables? ... In that way we can make but an approximate examination of the phenomenon, but one easily sees that there will be cases where, to examine the question in depth, it will be necessary to pass from a finite to an infinite number of variable elements).

admiration that has been further increased after my latest arrival in America. Your civilisation and your entrepreneurial spirit have been capable of conquering an entire continent, creating out of nothing as by magic marvellous cities such as this one we find ourselves in today [Houston]. They have grown up in a few years, and are not only equipped with all of the modern conveniences that render life more pleasant, but have also achieved an elevated level in intellectual and spiritual life . . . You have constructed, from the foundations up and very quickly, institutions and universities where everything is organised to respond to the needs of the present, without the weight of any relic of the past'.

5.2   From Neutrality to Intervention

With the American journey to Rice Institute we have arrived to the years before the outbreak of World War I, which would find Volterra decidedly on the side of interventionist positions. It should be noted right away, however, that Volterra was not a warmonger. Indeed, he had sensed the danger of a European war as early as 1911–1912, when Italy undertook the conquest of Libya and nationalist Serbs, supported by the Russians, accelerated anti-Turk and anti-Austrian activities in the Balkans.[5] In private, he made no secret of his feelings and was strongly against the colonial exploits in Libya. He feared a destabilising effect on the situation in Europe and the Balkans, and thus came to find himself taking positions that were different from those supported by public opinion in Italy,[6] and even from that of people close to him, such as his cousin Roberto Almagià (son of his uncle Alfonso and a university professor of geography), and his colleague Orso Mario Corbino. Corbino would come to play an important role in Italian physics in the mid-twentieth century; Volterra himself, together with Blaserna, had 'called' him to Rome in 1908.

An evaluation of the political profile of the supporters of the Libyan enterprise played a part in determining the attitude taken by Volterra. First among them were those who represented the economic interests of the Bank of Italy, which played a leading role in Catholic finances. However, the decisive element was a consideration of international public opinion. Volterra regretted the low opinion of Italy held abroad, particularly in the United States, and believed that an invasion of Libya – commonly held to be an unjustified aggression against a defenceless country – would further contribute to the disrepute into which Italy had fallen in vast sectors

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[5]See Volterra's letter of 17 October 1912 to his wife, quoted by Judith Goodstein in *The Volterra Chronicles*, op. cit., p. 175.

[6]See E. Ragionieri, 'La storia politica e sociale' in *Storia d'Italia Einaudi*, vol. 4, t. III, pp. XIV, 1665–2483 (Torino: Einaudi, 1976). The backing given by the great powers to Italian aspirations, the fear of damage to Italian colonial interests following the agreement between France and Germany regarding the crisis in Morocco, and Giolitti's pursuit of the Catholic consensus turned the tide of public opinion in favour of the initiative.

of public opinion in other countries. It is precisely the different international context that explains the new attitude he took when the world war began.

On 28 June 1914 in Sarajevo, the young Serbian nationalist Gavrilo Princip assassinated Archduke Franz Ferdinand (heir to the Austro-Hungarian throne) and his wife Sophie. In the space of a few short weeks, Europe was in flames, and Italy found itself divided between those who favoured neutrality and those who instead saw in the conflict an opportunity to complete the process of the Risorgimento, acquiring the territories of Trento and Trieste. The question was who to side with. The traditional anti-Austrian sentiment, whose roots lay in the Risorgimento, was strongly felt; on the other hand, there was the pledge to the Triple Alliance that Italy had signed in 1882 with Austria and Germany, and which had been renewed decade after decade up to as recently as 1912. For the time being, in August 1914, Italy declared itself neutral on the basis of article 3 of the treaty of alliance (invoking a lack of *casus foederis*, that is, its not being obligated to intervene because the attack on Austria was not unprovoked) with a declaration that was more or less accepted by the other Central Powers, who were well aware of the orientation of Italian public opinion. In the meantime, however, both sides involved in the conflict put Italy under an enormous amount of pressure. The strength of the nations of the Triple Entente – especially of France – pushed for Italy to enter into the fray alongside them, opening a new front along which to confront the enemy. For their part, the Central Powers did their best to make an argument that was just as convincing, at least in favour of continued neutrality.

Volterra came out immediately against joining the Central Powers. He was clearly on the side of a 'just' war and an alliance with the Entente. The legacy of the Risorgimento surely played a role in his interventionist stance: his personal and professional fortunes had gone hand in hand with those of the Kingdom of Italy, which had celebrated its first successful 50 years in 1911. Volterra identified with the Kingdom. He felt himself to be an integral part of its ruling class and authorities. In addition, in keeping with his upbringing as a non-practicing, assimilated Jew, he credited the monarchy and the state created by the Risorgimento with the defeat of papal power and the emancipation of the Jewish community. However, the decisive factor for his orientation, as we have said, was the drawing of enemy lines between factions. Volterra felt himself closely tied to the powers of the Entente, especially France, where the principal nexus of his scientific relationships were located. His closest and most esteemed colleagues were members of the academic world in France, and French publishers were responsible for disseminating his research results and ideas outside of Italy. In September 1924, when France had already entered the war, Volterra expressed his solidarity with his 'Latin sister nation' to his colleague Gaston Darboux, not hesitating to use expressions that were quite strong. Regarding German conduct during the war, he wrote of 'abominable crimes' and 'barbaric acts': 'permit me to tell you that my thoughts are always turned with the deepest attachment to you, to the teachers, colleagues, friends I have in France. Your great and noble nation is fighting for the cause of justice and civilization. All my wishes are for the success and the triumph of France. The act by which the two emperors [Franz

Josef I of Austria-Hungary and Wilhelm II of Germany] unleashed war and
destruction in Europe is viewed by me and by the majority of my countrymen,
as an abominable crime. The countless barbaric acts that the Germans have
committed during the war have only increased the horror and indignation felt in
a first moment. In my opinion Italy must take its place next to her Latin sister,
France, and its allies against Austria and Germany. This is its role and mission.
It should not be dismissed. I hope with all my heart that this will happen'.

The exchange between Volterra and Darboux is not an isolated example. The
intellectuals of the countries involved in the conflict actively participated in a
debate that was particularly painful, and often cut through ties that were long-
standing. Their intevention was solicited by the political world in order to sway
public opinion in one direction or another. In autumn 1914, with the circulation of
the 'Appeal to the Civilized World' signed by 98 professors of Germany was
circulated, there was a decided intensification of discussions concerning the
pacifism and internationalism of scientists, and whether or not it was their duty
to participate in the destinies of their respective nations. Volterra received a copy
from his German colleague Otto Staude. The document was a vigorous defense of
patriotic commitment and German motives against what were defined as
'distortions' on the part of Western propaganda. Among those who signed was
Felix Klein, with whom Volterra had a particularly cordial relationship. Other
signers included seven Nobel laureates for chemistry and physics! There were
also signatures of scientists such as Haeckel, Ostwald, Planck, Röntgen and
Wassermann, and humanists such as Dils, Morf, Wilamowitz, Wildebrand and
Wundt. Volterra's reaction is easy to imagine. How was it possible to attempt to
legitimise the violation of neutrality in Belgium and the retaliation of Germany
troops against civilians there? Even more important, he found the racist tone of
the manifesto unacceptable, especially declarations such as 'Those who have
allied themselves with Russians and Serbs, and who present the world with
shameful spectacle of inciting Mongolians and Negroes against the white race,
have the very least right to portray themselves as the defenders of European
civilisation'.

The manifesto signed by German scientists constituted a rupture of interna-
tionalism and solidarity within the world of science, and provoked an analogous
and opposite movement in the countries of the Entente. In mid-October, Volterra
received a request from Emile Borel for Italian material to publish in the *Revue
du mois* to inform the French public about the negative reaction of the Italian
people. Volterra's reply confirmed his full support of the cause of the Entente:
'... at the distance of a month I can only confirm what I wrote then, that is, my
best wishes for the success of France [and] my deepest sympathy for your noble
nation, which is fighting for justice and liberty and for the cause of civilisation
against the violence of a most brutal and hateful imperialism. I told you that
Italy's role is, in my opinion, to unite with the Triple Entente. I may add now that
confidence in that the union will take place has only increased since the sympathy
for France, England and Russia has grown among the Italians. On the other hand,
the conviction that all of our interests, from a moral as well as political point of

view, are in opposition to Austria and Germany has begun to consolidate. . . . You are perfectly right to be sure of the success of the cause of France and her allies. The whole world is against Austria and Germany. I am among those of us [in Italy] who are most eager to abandon neutrality, but I have no doubt that even those who are a little less impatient cannot fail to have the same hopes and aspirations as I've had since the beginning of the war. Italy, France and its allies must be united against the enemy who has committed the crime of unleashing war and wanting to enslave Europe'.

In Italy as well there was an organising movement to create or reorganise various associations for intellectuals in an anti-German key. In the month of December, the economist Antonio De Viti De Marco – a strong opponent in previous years of the policies and attitudes of Giolitti, and a colleague of Volterra's in the Accademia dei Lincei – asked him to support a manifesto that he intended to address to the intellectuals of all the nations of the Entente. Volterra agreed, expressing his hope, however that the list of those to receive it would be further expanded; he was thinking of the United States – which had not yet entered the war – and other neutral countries. He also suggested that the initiative follow the lead of 'the publications issued by the English men of science and by the French faculties in opposition to the letter of the German intellectuals, of which I disapprove. I maintain that many colleagues would willingly subscribe to this point of view'. At the same time, he wrote to the secretary of the Royal Society to try to give an organisational base to the intellectual movement. This is part of a letter that he wrote to the English physicist Joseph Larmor: 'that the war has derived from a wilful and premeditated aggression by Germany and Austria is shown by all of the documents that have appeared. I am pleased that such ideas are shared by the great majority of my compatriots, and I have no doubt that they will serve as a criterion of behaviour for Italy, whose intervention I hope will lead to noteworthy consequences. I am quite pleased to usher in 1915 with the wish that the ties between our two countries become increasingly closer.

Volterra's patriotism cannot be understood without understanding the climate that prevailed in Italy. Even in this context it is still difficult to relive the intensity of the idealist movement. The following passage taken from a letter by Volterra to his wife Virginia, written shortly after May 1916, when Italy had already entered the war, is expressive of what everyone was thinking: 'what are shame that our sons are not yet old enough to go to war!'[7] (Volterra's eldest son, Edoardo, was only 12 at the time).

Volterra's interventionism was not shared by everyone. The Italian intellectual world was not as unanimous in its opinion as his letters to foreign correspondence made it out to be, and opposition to the war involved influential figures (both inside and outside science), such as Tullio Levi-Civita and Benedetto Croce.[8] There were

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[7]This passage is cited by Goodstein in *The Volterra Chronicles*, op. cit., p. 180.

[8]On the particular attitude towards the war of the mathematical world, see Angelo Guerraggio and Pietro Nastasi, *Matematica in camicia nera* (Milan: Bruno Mondadori, 2005).

calls for caution and signs of bewilderment even after Italy entered the conflict. At the end of a meeting of the board of directors of the SIPS at Bonaldo Stringher's house, in September 1915, Castelnuovo wrote to a friend: 'we also discussed Reina's proposal to hold a partial or even full congress, or at least an administrative session of the Society. Once we had weighed the advantages and disadvantages, we had to admit (Stringher even more than the two of us) that the latter outweighed the former. . . . Any form of meeting might involve talks that, in a moment like this one, could not have a purely scientific character, but would necessarily cross into political territory . . . The atmosphere, even among scientists, is not as serene and harmonious as might be desired. Any spark could ignite a fire. And it is good thing if there is no trace of dissent in official proceedings of the Society. Thus we agreed in considering it appropriate to put off all society meetings until better days'.

In the autumn and winter of 1914, the Italian government was still uncertain about what to do, and continued to negotiate secretly with both the Central Powers and the Entente in order to guarantee the best possible territorial compensations when the war was over. In short, the interventionism of many intellectuals like Volterra had to come to terms with those in favour of neutrality, the pacifism of the socialists, and the cautiousness of the government itself. Even those in favour of intervention were a composite group which even included those who distinguished between Austria and Germany – unlike Volterra, Castelnuovo and Somigliana –, given that the organisation of the German systems of culture and science were still greatly admired. It is not surprising that when Italy entered the war, in May 1915, it did so against Austria, while it would remain 'non-belligerent' against Germany for a long time.

Volterra was no novice to politics. In earlier chapters we described his debut in 'academic politics' in Italy under the guidance of Ròiti, and under that of Mittag-Leffler with regard to international relations. His political horizons expanded at the time of his transfer to Rome, when he was co-opted by Giolitti's technocracy, finding a new political mentor in Bonaldo Stringher. Volterra's youthful conservatism had been oriented towards the 'centre', following Giolitti's overtures – whether judged real or timid and contradictory – to the socialist world and the working masses. Then he had been named senator. The breadth of his vision of the relationship between science and economic and social development was clearly expressed in his inaugural lecture of 1901 and in the constitution of the SIPS. Now, with the war, a new dimension was added to his political experiences: militancy. Interventionism required participation in public demonstrations and speeches from tribunes, which for Volterra, then in his 50s, was a novelty. But it was a novelty in which he had no qualms about involving himself, as shown by the letters he wrote to his wife in the early days of May 1915, when the celebrations for the anniversary of the departure of Garibaldi's Expedition of the Thousand from the rock of Quarto turned into those 'glorious days' extolled by D'Annunzio, in which Volterra took part.

In the meantime, the Italian government had arrived at the realisation that the Central Powers had no intention, in case of victory, of guaranteeing territorial compensation to a neutral Italy. It thus stipulated a secret agreement, on the other front, with London. This way of dealing, the object of much and justified criticism

by post-Fascist historiographers, corresponded to a feeling that was quite widespread in Italian interventionism, and which we find clearly expressed in a letter of November 1914 addressed to Volterra by the Florentine physicist Antonio Garbasso: 'The sentimental side of loyalty to the alliance does not affect me when what is at stake is the interests of the country, and it should not affect His Majesty, who knows the history of his dynasty. It was precisely by breaking faith with sworn pacts that Victor Amadeus II won the crown. But I am terribly afraid that the honourable Salandra and the honourable Sonnino want to be honest to the very end. Poor Italy'. Garbasso's fears turned out to be unfounded: on 3 May 1915 Rome officially broke with the Triple Alliance, denouncing the violation of the agreements on the part of Austria. Shortly after, the 'glorious days' provided the necessary popular support for an intervention already considered a certainty even before the session in parliament that was supposed to approve the decision. On 19 May Somigliana, writing to Volterra and referring to the 'great session of Parliament' of the previous day, ended the letter with 'thus we are at war; long live Italy!'. On 24 May Italy effectively declared war on Austria.

5.3   The Office for Inventions and Research

The very day before war was proclaimed, Volterra renewed at the Ministry of War his 'request already made this past April' to be enlisted 'in technical or laboratory service or in another service of any form or nature whatsoever'. At the same time, he accepted inscription in that list of 'persons with technical competence' called for by a decree of 29 April. His request for voluntary enlistment was accepted within 2 months: while he carried on his activities on the front of propaganda, Volterra was named a lieutenant in the corps of engineers, and assigned to service in the Central Institute of Aeronautics, under the direction of Major Gaetano Arturo Crocco (1877–1968).

Volterra had been interested in aeronautics since 1905, and Crocco had been in contact with him since 1906, when he had begun to work as a designer of hydroplanes and dirigibles at the airfield of Vigna di Valle (near Rome). At the beginning of World War I, the Central Institute of Aeronautics was the most important organisation in the field of aeronautic research applied to the military, and would remain so until the creation in 1935 of the research centre for aeronautical construction in Guidonia (also near Rome). After having founded it and assumed its directorship in 1908, Crocco – a true pioneer in Italian aeronautics, and following the war, a professor of aeronautic engineering at the University of Rome – had set up a program to study the effects of the wind on the motion of dirigibles and airplanes, and calculate the trajectories of projectiles fired from them (Fig. 5.2).

Crocco could hardly believe his good fortune in being able to entrust the research in aeronautic artillery (a completely new field at that time) to a mathematician of the calibre of Volterra. What was involved was calculating the firing tables for the
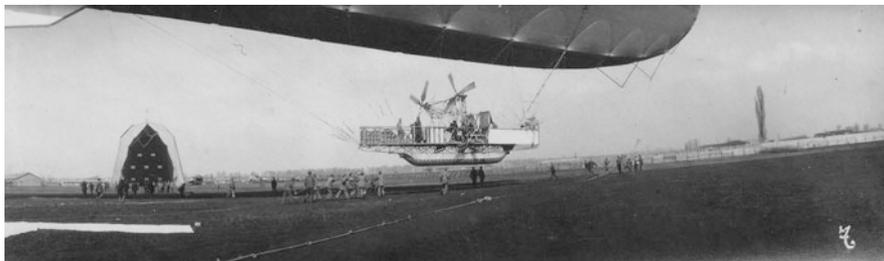
Fig. 5.2   At work on the dirigibles in the airfield at Campi Bisenzio (Near Florence)

65 mm mountain guns that Crocco intended to use to arm the dirigibles,[9] in spite of
the prevailing opinion that the dirigible would catch fire at the moment of firing.
Volterra dedicated himself to this question, with complete success, for more than a
year, addressing it with the classic attitude of the mathematical physicist. In the
paper published in 1916 in the *Rendiconti* of the Central Institute of Aeronautics,
entitled 'Metodi di calcolo degli elementi di tiro per artiglieria aeronautica'
(Methods of calculating elements of firing for aeronautic artillery), he first wrote
the differential equations that govern the general problem, solved them, and then
went on to setting out the methods for effectively constructing the trajectories. The
study is completed by the computation of the firing tables with instructions given
for the corrections to be made should, for example, the velocity of the dirigible
vary. Volterra and Crocco personally tested the correctness of the calculations by
climbing into the nacelle of the first dirigible from which a cannon was fired, on 6
June 1919, at the Florentine airfield of Campi Bisenzio.

   Volterra continued his work on dirigibles throughout 1916, taking personal risks
that earned him a promotion to the rank of captain and the *Croce di guerra* for
military valour.[10] Among other things, he proposed the use of helium in place of
hydrogen for inflating the dirigibles, thus minimising the risk of fire. The idea was
accepted only later and in other circumstances, but the merit of having advanced it
belongs to Volterra. Again in the field of ballistics, he took an interest in the work
undertaken by a young Sicilian mathematician to recalculate the firing tables for
warfare in the mountains. This was Mauro Picone (1885–1977) who, following the
war, would found and direct for many years the National Institute for Applications
of Calculus,[11] ultimately part of the *Consiglio Nazionale delle Ricerche* (CNR),
Italy's National Research Council. Ultimately based on an original idea of
Volterra's, this was also a development of the meteorological service: in speaking
of the Thalassographic Committee we mentioned the creation of a service for using

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[9]See Linguerri, *Vito Volterra e il comitato talassografico italiano*, op. cit.

[10]As late as 1920, he published his paper 'Osservazioni sul metodo di determinare la velocità dei
dirigibili' in the *Rassegna marittima e aeronautica illustrata*.

[11]See Guerraggio and Nastasi, *Matematica in camicia nera*, op. cit.

meteorological data in function of the requirements of air navigation. As the clouds of war had gathered, that service was transferred from Rome to Padua and placed under the direct control of the Supreme Command, constituting the *Sezione Presagi*, or forecasting department, of the Office for Aeronautical Services. Its job was that of communicating to aviation squadrons the weather conditions in the areas where air raids were planned. This early forecasting department was the beginning of what would grow and develop to become today's Meteorological Service of the Italian Air Force.

While Volterra was involved in these military activities, on 4 March 1916 his mother passed away. The pain of her loss was especially hard to bear because of the special place she had occupied in his life. To be sure, his memories went back to the sacrifices she had made when mother and son found themselves unexpectedly without any means of economic support and had received hospitality only in Florence, in the home of uncle Alfonso. Then he must have thought about the care with which Angelica had attended to his early education, and also to the years spent – alone – in Pisa and Torino, and to the support that, even recently, she had always given to Vito's ambitions. He mentioned the death of his mother in a letter to Mittag-Leffler written in May 1916, which, on the other hand, reiterated Volterra's complete commitment to his country and the requirements of the front. It was not a time for travel, not even for scientific purposes: '... you speak of a Congress of Mathematics in Sweden this year and a trip to Switzerland in the spring. I see that you do not have any idea of the mood in Italy. This is not the time to travel. All our thoughts are turned to the war we are fighting with great enthusiasm alongside our allies, and we think of the approaching moment of victory against our enemies. We are sure of victory and we hope for a happy future for our country, which was reluctant to place themselves on the side of justice and freedom. I joined the army and I am an officer in the corps of engineers. My military and technical occupations in the aeronautic corps now account for all my activity. My knowledge of mathematics and physics are useful to me in this moment'.

In effect, as we might well imagine, Volterra did very little research in these years. He produced only few publications, and almost all of these were occasioned by public events. To go back to the topic of the Thalassographic Committee, there was, among others, a speech for the inauguration in 1916 of the Institute for Marine Biology in Messina. Apart from this, there were only the publication of the text of some lectures and a few papers written or lectures given in the years just before the war, such as the long paper entitled 'Teoria delle potenze dei logaritmi e delle funzioni di composizione' (Theory of powers of logarithms and composition of functions) compiled in view of the 300 year anniversary of Napier in Edinburgh. Volterra's commitment to making his scientific knowledge available is quite generous, but his greatest contribution in these years was yet to come, and would be carried out on the organisational front. What began as the reorganisation of Italian research for applications to war, and its coordination with that of allied nations would lead, in just a few years, to the creation of Italy's National Research Council.

In spite of the immediate involvement of the world of science in the sphere of politics and ideology, and a widespread awareness of the differences in strategies and technologies between the war then underway and those of the past, the institutional implications of the new relationship between science and war were only gradually and slowly perceived. The turning point came with the attack of poison-gases launched by the Germans at Ypres on 22 April 1915 against an Algerian division of the French army. In open violation of the Hague treaties of 1899 and 1907, the German troops used chlorine, which attacked the mucous membranes of the nose, mouth, throat and eyes, causing immediate asphyxiation and blindness. Laymen and experts alike grasped the relation between the new chemical warfare and the technological advantage gained by Germany because of its particular scientific-industrial organisation. The French mathematician Paul Painlevé, who would become Prime Minister, characterised German science as a 'gigantic enterprise where people … were determined to make the most formidable killing machine that has ever existed'. The slogan 'do like Germany' was repeated almost to the point of obsession.

The trauma that followed the attack in Ypres drove the Entente nations to reflect seriously on the technological dimension of the war.[12] This increasing awareness meant the passage from a policy of inventions that simply accepted the contributions from individuals, to one of stimulating research on the part of public entities. New organisations were formed to coordinate the actitivies of the triangle of *science-industry-armed forces*. In England, the Board of Invention and Research and the Munitions Invention Department were created in summer 1915. In France, the Direction des Inventions Intéressant la Défense Nationale, which replaced a previous commission for the examination of inventions of military interest, was constituted in November that same year; it had at its disposal university and industrial laboratories where research was carried out on anti-gas protection systems, the improvement of firing tables, sound location of the enemy's batteries and submarine detection. In both countries the academic world was directly involved: the Royal Society established a war committee, while in France various scientists (who Volterra knew quite well) assumed positions of responsibility, including Painlevé, who would be the first to begin forming the *Comité interalliés des inventions* as a clearinghouse for bringing together the research activities of the allied countires and exchanging information regarding the results obtained. This was at the end of 1915.

In Italy, the first appeal for scientific-technological mobilisation and collaboration between researchers, industrialists and the military was launched on 9 July by the physicist Angelo Battelli, who had been co-director of the *Nuovo Cimento* together with Volterra and Felici, and who was one of the founders of the Italian

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[12]A great deal has been written about this subject. The interested reader can refer to L. Tomassini, 'Le origini' in *Per una storia del Consiglio nazionale delle ricerche*, I, Raffaella Simili and Giovanni Paoloni, eds (Rome-Bari: Laterza, 2001) and the bibliography therein. The quotation by Painlevé given earlier is taken from that essay.

Physical Society. The mobilisation of scientists that Battelli proposed was supposed to have been promoted by the government, with the establishment of either an agency or a special office set up in the Ministry of War.[13] Two days later, on 11 July, an article by the chemist Federico Giordano referred to a similar proposal conceived in the Politecnico di Milano and developed by Member of Parliament Giuseppe De Capitani d'Arzago, who had ties to both Prime Minister Antonio Salandra and the industrialists of Milan. De Capitani d'Arzago's proposal differed from that of Battelli in being openly private sector, as shown by its being intended to 'relieve' Ministry of the managerial load of managing a mobilisation that already promised to be long and involved. From the point of view, it appeared to be perfectly suited to Salandra's policies, who wanted his cabinet to be marked by a liberalist option in various sectors that involved the economic aspects of the war (ranging from propaganda, to assistance, to provisioning). Battelli's proposal went no further, having become lost in bureaucratic conflicts over the organisation of the undersecretaryship for Arms and Munitions,[14] while the initiative born in Milan went forward immediately, with two meetings at the Politecnico di Milano and the establishment of a national committee for inventions for war and the approval of its by-laws.[15]

Although establishment of the Milan committee was inspired by similar experiences abroad, in reality Giordano's presentation of it showed a lack of understanding about what was going on elsewhere. The fundamental tasks that the Milan committee set for themselves relied heavily on individual inventors: 'to discern among a large number of proposals expected to arrive those which are worthy of consideration; to provide a scientific and possibly experimental basis for proposals that are interesting but only superficially developed or verified; to serve as a link to military administration'. In contrast, for the structures created in France and England the principal purpose was not so much that of evaluating the latent resource that the inventors represented, whose contributions were showing themselves to be rather disappointing, as much as operating within a perspective of promoting and directing institutional research (in both private and public sectors). The inadequacy of the Milan committee was immediately evident in the relations

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[13]Battelli's appeal appeared in the *Giornale d'Italia* of 9 July 1915, the same day on which the under-secretaryship for Arms and Munitions was instituted in the Ministry of War. Giordano's article, which we discuss immediately after that of Battelli, was instead published in the *Corriere della Sera* on 11 July. The first newspaper, published in Rome, was certainly closer to ministerial circles, while the second, published in Milan, was the utmost expression of industrial interests. One of those called to participate in the Milan committee was Luigi Albertini, the director of the *Corriere*.

[14]Constituted on 9 July, the under-secretaryship would receive its by-laws only on 22 August 1915. The Central Committee for industrial mobilisation (which was one of its primary activities) met for the first time on 18 September. The world of industry always remained diffident about what it considered to be a form of creeping nationalisation.

[15]The two meetings took place on 19 July and 14 August 1915. Figuring among the members of the committee were Giuseppe Colombo (of the Politecnico di Milano), Giovanni Battista Pirelli, Carlo Esterle (of Edison), Luigi Albertini and Guglielmo Marconi. Giordano was named director.

with allied powers, but its internal structure was also inadequate. The bodies established in the other countries were state entities, and because only public bodies were allowed to participate in the Comité Interalliés, the Milan group was not admitted and was forced to resort to a subterfuge to send its director Giordano to the international meetings as a representative of the Italian government.

At the inter-ally level, another problem derived from the fact that Italy was only at war with Austria – not with Germany – and this led to only a partial sharing of efforts, inasmuch as Italy was not seen as an entirely reliable partner. A letter from Somigliana to Volterra mentions this problem, and also shows the bewilderment and cautiousness – this was spring 1916 – that were still widespread among intellectuals: 'Laura sent me your proposal to invite Hadamard to give a lecture in Torino. The idea is a very good one and I thank you. But there are some problems. I have spoken to Segre about it so that, as dean, he could send the invitation. But he raised many doubts. ... In conclusion, my impression is that he is worried about the idea that he might have to make a demonstration of something other than a proper welcome for the mathematician Hadamard. Unfortunately, that's what the environment in our Faculty is like ... The predominant concept is that we must live like we are in the Limbo of the holy fathers, ignoring the war, without any dislikes or likes for anyone, except for due respect for the Germans. Now frankly, I think that Hadamard would be coming to Italy for something more than a simple exposition of theories of analysis; and that bringing him into contact with these elements might lead him to arrive at an impression of our country that is not what we want him to have'. Naturally, Somigliana's interpretation of the reasons for Hadamard's visit to Italy were correct: he went to Rome to give a lecture on Cauchy's problem for hyperbolic equations, but also to step up relations with the scientific and military spheres in Italy. This is what he wrote to Volterra on the eve of his visit: 'My dear friend, Painlevé has just let me know of your desire to have a French professor in Rome, and that you would look favourably on me. I need not tell you how much I appreciate your invitation. ... The objections you understand, and I think you will have guessed them in advance. Not only, like everyone, am I able to think only about what concerns the war – and it is true that your idea issues from that, so this would not be an objection – but I have the joy and pride of collaborating on work of direct use to the Defense. I am attached to the new Department of Inventions, which is headed, as you know, by Painlevé and Borel. ... On the other hand, I understand along with everyone else how important a statement this is, for both countries, on scientific grounds, of their solidarity. I would very much like to reconcile this duty with the ones I just mentioned'. If there is any need to underline the drama of the moment, it is provided by the news that reached Rome just after Hadamard boarded the train to return to France. It was Virginia who opened the telegram that said that Pierre Hadamard, the eldest son of the French mathematician, had been seriously wounded in the battle of Verdun. He died just a few days later.

The climate and the feeling of the war changed around the middle of 1916, when it began to look possible that the United States might intervene on the side of the Entente. The sinking of ships in the Atlantic by the Germans, from that of the

Lusitania in May 1915 to that of the Essex and Sussex in March and April 1916, had stunned the American public and shaken President Wilson's conviction in neutrality. Thus a series of institutions were set up in the United States as well, in view of a possible mobilisation of science and industry. Their principal promoter was George Ellery Hale, the astrophysicist with whom Volterra had been in friendly correspondence since 1909, when Hale had been in Rome to give a lecture on 'Solar Vortices and Magnetic Fields'. It was under his impulse, and in preparation for a possible intervention, that the National Academy of Sciences established the National Research Council, which achieved major recognition for its political and institutional role[16] just as the electoral campaign was underway that resulted in Wilson's re-election.

In Italy during the same period, Volterra founded the Italian Association for Intellectual Understanding between Allied Countries and Friends, which involved intellectuals of diverse political and professional profiles.[17] His credibility extended beyond the confines of the scientific community, and he was by now visible as a public figure. The Association published the magazine *Intesa Intellettuale*, which was a significant point of reference for disseminating ideals, even if its activities mainly involved the area of publishing and the promotion of Italian books. In the second half of 1916, its members included Bonaldo Stringher, Maggiorino Ferraris and the publisher Formiggini, while Croce remained strongly critical and refused Volterra's explicit invitation to join. For Croce, it was an error to speak of 'German barbarism', and 'of all the errors that are the fruit of the moment, this wins the prize, because it is certainly the most grandiose'. In July 1916, under the aegis of the SIPS, whose president at the time was the anatomist and pathologist Camillo Golgi, the National Committee for Science and Technology was established, with the aim of fostering greater collaboration between the world of industry and that of science and technology. The Committee depended on the same industrialist circles that promoted the earlier committee for inventions, but the points of reference were now institutional and academic, and the aims more far-reaching. The driving force behind it would turn out to be Giuseppe Colombo, founder of the *Società Generale Italiana di Elettricità Sistema Edison*, Italy's Edison electric company, and one of the most authoritative figures in the Politecnico di Milano (Fig. 5.3).

On 28 August 1916 Italy gave in to pressure from its allies and declared war on Germany. Let's follow the news from those months. In October, Borel – in agreement with Painlevé – undertook a short mission in Italy, during which he met with Volterra who, for his part, went to Paris at the beginning of November. Successively he undertook a mission to the French front to study weaponry and technical innovations introduced by allied armies. Central to these meetings was a desire, shared by all the allies but especially felt by the French, to organise relations

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[16]See Helen Wright, *Explorer of the Universe: A Biography of George Ellery Hale* (New York: American Institute of Physics, 1994).

[17]Members of the executive committee included, among others, Marco Besso, Guido Castelnuovo, Antonio De Viti De Marco and Alberto Tonelli.

Fig. 5.3 A postcard to Volterra signed affectionately by his son Gustavo
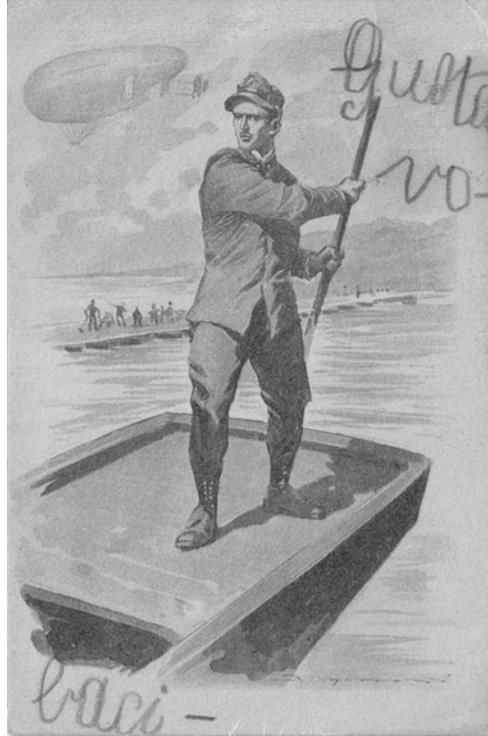
of scientific collaboration with Italy on an institutional basis. Without mincing words, Borel wrote, 'we . . . have organized very close relations with England. Our relations with Italy are less developed'.

On his return from France, Volterra prepared a detailed report for the Minister of War, who replied on 24 January 1917, inviting him to draw up a plan for the establishment in Italy of an office analogous to the French *Direction des Inventions*. This led to the institution of the Office of Inventions in the month of March, with Volterra named as director. Giordano remained the government representative to the Comité Interalliés des Inventions in Paris, but his activities and that of the Milan committee took a back seat. Working in favour of the new arrangement was the network of contacts that Volterra had in Italy and abroad and his formal position as an officer in the military (though temporary), which allowed him to move about with greater freedom within the organisational apparatus of the armed forces. From this moment on, Volterra focussed his attention on directing the new office, becoming increasingly the central pivot in the triangle of science-industry-armed forces.

His first step in guiding the Office of Inventions was the rationalisation of the existing procedures for examining inventions in order to make them less fragmented and to put the new office's function of coordination into effect. In a letter to the physicist Michele La Rosa written in spring 1917, he wrote:

'The studies that have raised an especially keen interest are those you have undertaken on an underwater microphone for signalling ships and submarines. ... With regard to the proposal for a device for firing projectiles underwater against submarines, which also seems noteworthy, the best thing would doubtless be for you to send us, in two copies, a new report and the relative drawings, without taking the previous [agencies] into account, as these have no relation with our Office'. In the months that followed, Volterra's activities proceeded along three complementary lines: the organisational consolidation of the new body, the broadening of its institutional function, and the strengthening of relationships of scientific cooperation with allies. Useful to these ends was the creation of a scientific 'branch' of the Office, whose job it was to 'direct the work of technicians and scientists intended to solve the most serious problems of the war itself as well as of the war industry'. Volterra's standing made it possible for him to obtain effortlessly the collaboration of leading figures such as Orso Mario Corbino, Raffaello Nasini and Federico Millosevich, and to thus contribute to the creation of 'special services' for research in chemistry, physics, mineralogy and engineering. At the same time, Volterra was engaged in a struggle with the Milan committee for inventions for war, and succeeded in obtaining from the Ministry a series of directives which in fact obliged the Milan committee to refer to the Office of Inventions for all relationships with military organisation, and to limit its activities to the sectors of mechanics and electrotechnics (where their industrial interests actually lay). At the beginning of 1918, on the basis of the broadening of the activities carried out, Volterra proposed to the Minister that the name of the office under his direction be changed to the Office of Inventions and Research. His request was granted without further ado on 24 February 1918.

For Volterra, the question of international relations remained particularly relevant. The technical-scientific relationships between allied nations continued to be mediated by the Comité Interalliés in Paris, in which Volterra confirmed Giordano's role, although privately he expressed opinions about his work that were not very favourable. The fact was that he didn't consider this channel to be very important, and preferred to deal with the most sensitive questions personally, using his contacts with the leaders of allied institutions of military research, or assigning responsibilities and missions ad hoc. Towards the end of the war, he was able to make it so that the mathematician Emile Borel was sent to Rome as a French representative. In October 1917, he sent lieutenant Sborgi to London as an Italian representative. But it was above all on the front of relations with the United States, which entered the war on 6 April 1917, that his most important activities were carried out. As a connection Volterra made use of Giorgio Abetti, a young astronomer who was the son of the director of the astronomical observatory in Arcetri, outside Florence, who already before the war had been an assistant to Hale, and thus had a direct foothold in the heart of the American scientific organisation. It's a small world, and the American representative in France was the same Evans who had perfected his studies in 1910–1912 in Rome under Volterra, who had in his turn helped him to obtain an academic position at Rice Institute in Houston. In 1916, Evans had become a full professor in Houston,

where he remained until 1934, when he transferred to Berkeley. Thanks to Evans and Abetti, Volterra stayed informed about Hale's projects, which already 'galloped' beyond the scenarios of war.

By this time, it was 1918 and no one could say precisely when the conflict would end. In any case, the entry of the United States had changed the destinies of the players. For the countries of the Entente, all signs were positive. It was only a question of time. They began to think of 'after' and capitalise on the efforts undertaken and the benefits acquired from the experience of the war. It was July 1917 when, under express instructions from Hale, Abetti sounded out Volterra about the future of the Office of Inventions, asking if 'it had been his idea, when he had established it, that it should, after the war, become an institute intended to serve as a close connection between science and industry . . . This is similar to how Hale had instituted the National Research Council'. In 1918 European scientific organisations began to look at American ones with interest.

The first time we mentioned American mathematics it was in connection to Chicago and the first instances of international congresses. That was 'only' 1893. And yet how much water – in just a quarter century – had passed under the bridge! With 'Chicago 1893' we left the United States while they were looking for an eminent European researcher willing to transfer to the American continent, but contenting themselves if a mathematician of the calibre of Felix Klein (or some other brave soul who was willing to undertake such a tiring journey) would come for over for a few weeks to test the terrain and 'sow some seeds'. We have already had a glimpse of what was happening on the other side of the Atlantic during Volterra's trips in 1909 and 1912. There is no other way to explain his sincere admiration for the American capacity for innovation and their way of structuring things, which Volterra could see for himself. However, the new aspects that emerged during and after the war were expressive of the kind of change that could justifiable be called epoch-making. Science, which up to just a few decades earlier had been a strictly European domain, carried out primarily in German and French, was revolutionised by the new organisational concepts of the United States, and soon by its results. The most innovative ideas began to arrive from the other side of the ocean, long before the exodus of scientists of Jewish descent from Europe.

The institutional core of the process of transformation of American scientific policies was the National Academy of Sciences in Washington, founded in 1863 during Lincoln's presidency, but still at this time in search of a role in American society comparable to the one of the great European national academies. A tangible expression of this aspiration and of its first results was the inauguration of the grand headquarters built in 1913 in the nation's capital, thanks to the contribution of the Carnegie Foundation and ongoing fund-raising by Hale and other academy members. In charge of international relations, Hale suggested passing a resolution in which the Academy declared its willingness, in case of a break in diplomatic relations with Germany, to consult with the government and coordinate the scientific community's participation in the war effort. There was no lack of opposition within the scientific world on the part of those who felt passed over and feared the

possibility of a hegemonies of specific academic, disciplinary or industrial components. Initially, some of the most technologically advanced sectors of industry were also wary of the kind of coordination proposed, convincing President Wilson instead to form a committee of inventors led by Thomas Alva Edison, which was supposed to collaborate with the navy through the Naval Consulting Board.

In April 1916, the idea of Hale and the National Academy of Sciences was presented to Wilson, who, finally convinced to the usefulness of the proposal, gave his consent. The work of planning could continue, but in the strictest confidence: with elections coming up soon, Wilson absolutely did not want to alienate the strong component of those favouring neutrality and isolationism. We mentioned earlier that the conclusions of the specially formed committee, presided over by Hale and constituted within the Academy, had led to the creation of a new body that its creators named the National Research Council.[18] It was now June 1916. The next months of the summer were spent in contacting and applying pressure to obtain the practical and formal green light from the president and broad support from the world of industry. Then, Hale and the president of the Academy travelled to England and France. This was the final test, which ultimately convinced them of the timeliness of what they were organising. In February 1917, after the rupture of diplomatic relations with Germany, the Council of National Defense officially requested the collaboration of the National Research Council. This was the beginning of frenzied organisational work in which, along with that of Hale, a significant role was played by the physicist Robert Millikan. The National Research Council obtained private and public funding – those of the private sector much greater than those of the public sector – especially from the Carnegie and Rockefeller Foundations, although, during the months of war, funds and administrative personnel showed themselves to be insufficient for the objectives fixed: a census of personnel, equipment and scientific activities underway; collaboration with universities and research institutes both public and private; promotion of studies linked to national defense and the coordination of related research projects; and the creation of a centre for gathering and disseminating scientific information.

The armistice arrived sooner than expected, before such programs could be completed. However, the National Research Council immediately represented an important change. In the United States, researchers and industrial leaders already worked in close contact to define joint programs between universities and businesses, but there was no institution for promoting and coordinating initiatives. Now, this synergy found a natural channel, and industrial leaders and professors no longer had to rely on chance meetings at the faculty club for the exchanges needed to augment and direct research activities.

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[18]See Wright, *Explorer of the Universe: A Biography of George Ellery Hale*, op. cit.

Chapter 6
From War to Peace: Italy's National Research Council

6.1 The Reorganisation of Science: The Ideas Come from the United States!

As we have seen, the opportunity to create the National Research Council was provided by the war. Nevertheless, even before the armistice, there were some who believed that its primary focus ought to be shifted from military activities to those of industry. Hale wanted the new agency to become a permanent organisation for promoting and coordinating scientific work, in collaboration with industrial complexes and their needs. In 1919, he wrote: 'The Academy organized the National Research Council ... with a view to stimulating the growth of science and its application to industry, and particularly with a view to the co-ordinating of research agencies for the sake of enabling the United States, in spite of its democratic and individualistic organization, to bend its energies effectively toward a common purpose'.[1]

The first important contribution of the National Research Council was support for a research base in the field of chemistry and physics, by means of a grant program financed by the Rockefeller Foundation, which, inaugurated in 1919, would turn out to be particularly important in stimulating 'curiosity-driven' research and the training of young researchers. At the same time, Hale set to work on an international perspective.

We mentioned earlier that in 1917, through Giorgio Abetti, Hale had already sounded out Volterra's intentions for extending the organisational structures tested during the war into the post-war period. That this idea included the possibility of making inter-ally cooperation permanent is made clear in his correspondence with the English astronomer Arthur Schuster. Hale wanted to

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[1] From a memorandum entitled "Confidential: The Origin and Purpose of the National Research Council", dated 21 May 1919 (box 41, Millikan Papers). See Ronald C. Tobey, *The American Ideology of National Science, 1919–1930* (Pittsburgh: University of Pittsburgh Press, 1971), p. 52.

extend the victories earned on the battlefields to the new situations created after the debris of the war had been cleared away; he wanted to continue to develop the transfer of technology that had begun to render extremely important services during the war, making it a permanent instrument for social progress and economic supremacy. Taking as a model his own National Research Council, Hale suggested to his correspondents the creation of an organisation that would go beyond the experiences of international collaboration undertaken up to then, and could stimulate and coordinate the great scientific undertakings that required the combined efforts of scientists from different countries. Up to then the most significant experiments in this sense were those carried out by the community of astronomers. Hale had praised their international dimension from the time of the congress of astrophysics held in Chicago in 1893 (which took place in parallel with that of the mathematicians).

The new international agency was supposed to be constituted under the aegis of the most important national academies: the Royal Society of London, the Académie des Sciences of Paris, the Accademia dei Lincei of Rome and the National Academy of Sciences of Washington. Hale then proposed the constitution, within this Inter-Allied Research Council, of international unions for the various disciplinary sectors, taking as an example the International Solar Union which had been in operation since 1905. His proposal was formalised in April 1918 and unanimously approved by the National Academy. This opened the way for a series of inter-ally conferences for programming and organisation, in which Volterra participated as both a delegate of the Accademia dei Lincei and as director of the Office of Inventions and Research.

The first conference took place in London from 9 to 11 October 1918, just a few weeks before the end of the war. With Burlington House as the venue, the Royal Society hosted delegates from Belgium, France, Great Britain, Japan, Serbia, Italy and the United States. The proposal to constitute the Inter-Allied Research Council, as Hale had formulated it, called for each nation to first of all establish its own 'national research council', effectively transforming the agencies created during the war into bodies for coordinating the world of research and that of industry, and for directing their results towards the war effort. The various 'national councils' would then give rise, through the involvement of their delegates, to an 'international council'. The proposal was accepted, but there immediately arose the problem of the participation of German scientists (and their allies).

All those who took part in the London conference agreed that these must be excluded from the new agency. Scientific internationalism had been broken. The appeal 'to the peoples of civilised nations' that an important group of German intellectuals had signed, denying German responsibility for the outbreak of the war or any criminal conduct, hung like a stone around their necks. The hawks were the French and Belgians (it isn't difficult to understand their position, given what the war had meant for those countries in particular): these two delegations insisted adamantly that Germany be excluded from all future international organisations, and that this specific clause be inserted into the negotiations for peace. Their request was only somewhat mitigated by the usual compromise of the institution of a

commission charged with studying the concrete ways of creating an inter-ally council; the commission included Picard, Hale, Schuster, Volterra and the Belgian Georges Lecointe. It seems natural that Volterra would participate at this level, given his contribution to events leading up to this, but in the decades to come it wouldn't always be like this; the occasions when Italian science is represented in the highest-level international bodies will become increasing more sporadic. The London conference closed with the convocation of the next meeting, to be held in Paris from 16 to 29 November.

During this brief interval, just over a month's time, the armistice was signed and the projects designed for scientific cooperation after the end of the war became more relevant than ever. The Paris conference led to the founding of the International Research Council, or IRC, with French mathematician Émile Picard as president, English astronomer Arthur Schuster as secretary general and temporary headquarters in London. The commission nominated in October was transformed into a temporary executive committee, charged with drafting the by-laws for the organisation and preparing the successive conference in Brussels, planned for July 1919, at which time the new international council was to be officially inaugurated. In Paris, the Italian delegation was composed of Volterra and several other colleagues from the Accademia dei Lincei and the Office of Inventions and Research. The Paris conference also included the drafting of a list of countries that were eligible for membership in the IRC (those of the Entente and their allies) and a second list of those nations that could be admitted upon request (essentially those nations which had remained neutral).

Hale once again underlined that the primary aim of the IRC was that of effectively promoting international cooperation in the field of science (and not just the "simple" organising of congresses). The spirit of the American initiative was summed up in this way: 'It is very satisfactory to see a group of scientific men gathered from all parts of the world, working together in the most cordial manner and with the most complete disregard for national boundaries. Such men, in the course of time, are bound to have wider influence in national affairs, because the value of their knowledge and experience is being increasingly recognised by the state. I therefore believe the encouragement and extension of international cooperation in research to be a favourable field of action for those engaged in the promotion of peace'.[2] These are words that demonstrate hope in the future, as is natural for a period just after a conflict as devastating as World War I. They are, however, words that are in potential contradiction with the idea of permanent, or in any case long-lasting exclusion of countries that were enemies on the battlefield. It would not be long before that contradiction came to light.

For the mathematical world, the analogous situation was only postponed by 2 years, until 1920, when the first international congress for mathematicians after

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[2]Helen Wright, *Explorer of the Universe. A Biography of George Ellery Hale* (College Park, MD: American Institute of Physics, 1994), p. 303.

the war was scheduled to take place in Strasbourg. The choice of the venue, desired by the French mathematicians, carried a precise message: Strasbourg is the city of the Alsace that the French had just reacquired with the signing of the treaty of Versailles, after its having passed to Germany in 1871. Its being chosen to host the congress reflected the anti-German attitude of the victors. The mathematicians of Germany and its allies defeated in the war were barred from participation, since they were prohibited from taking part in any initiative that referred to the IRC. It was this climate that had led the French academies to force many of their German members to resign.

It was this same climate that led Volterra to edit his *Saggi scientifici* in a 'special' way. This book, published in 1920, was an anthology of some of his lectures and papers given at the congresses of Paris in 1900 and Rome in 1908, which we discussed earlier in Chaps. 3 and 4. All references to German mathematicians have been expunged from the texts as they are published here. Klein and the Erlangen program simply no longer exist! They were simply cancelled – Klein had been one of the signers of the 'appeal' of the German scientists in 1914 – as were other mathematicians such as Fuchs, Neumann, de Bois-Reymond and Kronecker. Volterra, a man of science and of politics, was also a man who 'took sides', and this shows that he had not yet forgotten the feelings that were evident in some of the letters quoted in the previous chapter. He was not alone: Picard concluded the Strasbourg congress with the warning that 'to forgive certain crimes would make one an accomplice'.

Volterra would have a hard time, even with the passing of years, taking the edge off the anti-German feelings that arose out of the war. In 1922 Levi-Civita, with whom his relations were most cordial, invited him to take part in a conference on fluid dynamics that he was organising with the German mathematician Theodore von Kármán,[3] and which was one of the first attempts at resuming collaboration with German researchers. Volterra's reply was unusually terse: 'I can neither speak nor attend'. Still to come in Germany were Hitler and the racial persecutions, and in 1937 Volterra would refuse to authorise a German translation of his works on population dynamics.

Scientific internationalism became a mere memory of the past. For the mathematical world a long struggle began over whether or not to re-admit German colleagues into international collaborations, a struggle that would conclude only with the International Congress of Mathematicians in Bologna in 1928. That time it would be the German mathematicians who were divided. There would be those, like Hilbert, who would take in good part the hand offered him by the colleagues of the victorious nations, but there were others, like Brouwer and Bieberbach, who had no intention of pardoning an attitude that offended 'the memory of Gauss and Riemann'.

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[3]For this episode, see Giovanni Battimelli, 'Senza alcun vincolo ufficiale: Tullio Levi-Civita e i congressi internazionali di meccanica applicata', *Rivista di Storia della Scienza*, II Ser., 4 (1996): pp. 51–80.

6.2   From the International Research Council to the Italian National Research Council

Back in Italy after the Paris meeting, Volterra immediately set into motion the most pertinent initiatives in order to pave the way in Italy as well for the future that seemed to await the international scientific community. At stake was the wave of international cooperation, which was absolutely not to be missed. Also at stake was the possibility to develop initiatives begun with the Italian Society for the Progress of Science, creating a new centre for Italian research that would overcome the fragmentation that had been leading to increasing paralysis. These were objectives that were coherent with those that formed the basis of the foundation of the Society, but also developed them further. The Society had been founded to create a place where scientists from different areas could meet and debate, and to put pressure on political forces to recognise the social and economic importance of the scientific movement. Now the most important item on the agenda was no longer the creation of an occasion for 'assembly', but rather the timeliness – at least this was Volterra's hope – of harvesting the fruit of the mobilisation of the previous years and constituting some kind of 'command centre', taking advantage as well of the persuasive force of the international movement.

On 18 December 1918 Volterra wrote to the Minister of Industry regarding the possibility of a new role for the Office of Inventions and Research, now that peace had been achieved: 'The Office of Inventions was instituted during the war and its first responsibility was that of examining inventions for war; but quite soon the field of its actions broadened and the office dealt with research of a technical, scientific and industrial nature, directing its activities to the search for new sources of wealth in the Italian land and was a consulting agency for technical questions of various ministries and departments in addition to that of War and the Navy (the ministry of Transportation, that of Industry, the commissariat for combustibles, the Ministry of Agriculture), so much so that the name of the Office was changed, and became Inventions and Research. Working for the Office are illustrious professors and distinguished technicians, in whose name as well as my own I ask Your Eminence that the Office be allowed at this time to pass from the Commissariat of Arms and Munitions to being under the supervision of the Ministry of Industry'. 'Arms and Munitions' was in the process of being dismantled. The winding-up of the various services had been entrusted to the undersecretary of the Treasury, Ettore Conti,[4] who made this note in his agenda: 'Orlando showed me a decree about which there

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[4]Senator, shareholder and administrative advisor for numerous companies and industrial groups (including Montecatini and Edison), Ettore Conti was one of the most authoritative figures of the industrial bourgeoisie in Lombardy, and was behind some of the most important operation of conversion following the first world war. Conti actually did leave the Office of Inventions and Research out of the first dismantling operations, as he promised Volterra. However, the difficulties encountered during the creation phase of the National Research Council later forced him to go forward with dismantling the Office as well.

was no doubt, already signed by His Majesty, for the constitution of a High
Commissariat to re-engage the two previous Ministries of Arms and Munitions
and Aeronautics and regulate the passage from war to peace; and a second decree
for my nomination as High Commissary'.[5] Relying on the support of Conti, who
assured Volterra that the Office he directed was not for the time being involved in
the project of winding-up, so that he had the time he needed to transform it into an
agency suitable for the post-war period, Volterra dedicated himself to promoting
the creation of the 'National Research Council'. In reports presented to the
Accademia dei Lincei in November 1918 and January 1919 he identified the Office
of Inventions of Research as the nucleus of the council to be formed, with the
important integration of new national laboratories for experimental research, based
on the example of what had been done in other countries (in France in particular).
Volterra and Crocco, who flanked him in this venture, intended the Institute for
Aeronautics to form the first of these laboratories.

Between the end of January and the beginning of February, with the contribution
of others working in the academic and industrial spheres, Volterra and Crocco
drafted a series of concrete proposals regarding the institution of the National
Research Council and the transformation of the Institute for Aeronautics. They
also prepared a preliminary decree for the Prime Minister, Vittorio Emanuele
Orlando, to nominate a commission charged with drafting a decree to constitute
the new council. That commission was nominated on 17 February, with Giovanni
Villa as president (he would later become vice-president of the Council). In actual
fact, the decree concerned not only the nomination of the commission, but
introduced other details as well. It specified that the council to be constituted
would absorb several agencies already in existence (the Office of Inventions,
Committee for Science and Technology, the Committee for Chemical Industries
and the Institute of Aeronautics), and take advantage of the 'elevated cooperation'
of the Accademia dei Lincei. The commission included the Minister of the Treasury
Bonaldo Stringher and Undersecretary Ettore Conti, Volterra for the Office of
Inventions and Research, Crocco for the Institute of Aeronautics, Nasini for the
Committee for Chemical Industries, Senator Giovanni Battista Pirelli and Fernando
Lori for the Committee for Science and Technology, and Romualdo Pirrotta for the
Accademia dei Lincei. The aims and structure of the council were essentially those
outlined during the inter-ally conferences in London and Paris.

The project appeared to be well underway, with the commission beginning to
conclude its work, when on 23 June 1919 the Italian government underwent one of
its periodic crises. With Stringher ousted from his position as Minister of the
Treasury, one of the main links in between the government and the commission
was broken. The scientific coterie was also forced to momentarily slacken its
involvement in order to participate in the Brussels conference, at the very moment
when a certain degree of opposition to the idea of the new national scientific

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[5] See Ettore Conti, *Dal taccuino di un borghese* (Bologna: Il Mulino, 1986).

Fig. 6.1 One of the giant sequoias in Yosemite National Park, on a postcard that Vito Volterra sent to his children

laboratories was beginning to crop up in the academic world. Earlier, in March, a commission of the Lincei had raised questions about the constitution of the International Research Council, and later expressed an explicitly negative opinion with regard to the laboratories. Professional jealousies and vetoes are not surprising: a new post-war Italy was being constructed! It is almost natural that institutions already in existence would respond to these new developments with a conservatism that was motivated by the strong desire to maintain their own piece of the pie, and by fears that this would be eaten away by the new agencies.

The international conference in Brussels, from 18 to 28 July, led to the formal constitution of the International Research Council. Volterra gave his all to drawing up the rules that would govern the life of the new agency in keeping with what had already been decided in London and Paris: the definite exclusion of Germany and the constitution of international unions for the various disciplinary areas. After the approval of the by-laws, Volterra's membership in the executive committee was confirmed with his appointment as vice-president. Italy's contribution to the birth of the new council was therefore an important one: Italy was in the right place at the right time, among the leaders. Italy understood that there was much to be gained by collaborating with nations that were more advanced at the time. What's more, the existence of an officially recognised centre for international coordination provided further legitimacy for the Italian scientific community and increased the demand for its services internationally, offsetting the weak internal demand for coordination and promotion of research coming from both the universities and the world of industry (Fig. 6.1).

Hale was not able to take part in the Brussels meeting, absorbed as he was by a series of commitments in the United States that made it impossible for him to be away for the long period of time required for the journey. Instead it was Volterra who, after the Brussels conference, travelled to the United States, and especially to

California. This time he stayed for quite a while in Pasadena. Hale had written asking Volterra to keep him informed about his travels so that he could make sure to be there when Volterra arrived. The trip was thus planned months in advance. Abetti's letter is dated 13 March: 'I wanted to tell you that your visit here would be most welcome, and I believe that if you were able to come all the way here it would be a most useful and important thing. In addition to getting to know these colleagues of the West, who would be quite glad to meet you, you could visit the Mount Wilson Observatory, and get a more concrete idea of our scientific equipment, also with a view to giving our country a push towards something similar, even though the means are much more modest'. Abetti's words would turn out to be prophetic with regards to the reception that awaited Volterra in what would turn out to be his last trip to America, as well as with regards to the installation of several research instruments at the observatory in Arcetri after the young astronomer's return to Italy.

With the conclusion of the Brussels conference and Volterra's return to Italy, the work of the Villa commission quickly came to an end. The prime minister was sent a final report that emphasised the necessity of putting the nation's resources to the best possible use, and involving, to this end, government administration, and in particular the ministries concerned with economics and the military. The report maintained that the means for achieving this strategic objective was the creation of an centralised institute for research with nationwide responsibility, not one directly connected with universities, which would coordinate and promote both fundamental and applied research in Italy. The required funding– fully in kine with what had been allotted for analogous centres in other countries – was estimated at two million lire for the initial establishment and an annual budget of 450,000 lire, without excluding a priori the possibility of acquiring additional resources deriving from activities undertaken or from contributions.

Of course, at this point, however, there was a lack of political resonance with the government. Opposed by the new Minister for Industry Dante Ferraris (one of the major critics of the 'statism' of industrial mobilisation during the war), and not supported by Prime Minister Francesco Saverio Nitti, the proposal ran aground in the ministries. The impasse was fatal. Just a few months later, in June 1920, the Giolitti government underwent another crisis, and the political climate changed drastically. The new Minister for Public Education was Benedetto Croce, who already during the war had shown himself to be contrary to projects for intellectual and scientific cooperation, and now post-war had become the champion of the dismantling of the apparatus erected for the mobilisation of industry, of which he tried to stamp out even the memory. In Chap. 5 we mentioned the dispute between Croce and Enriques in 1911: Croce was against scientific culture, in which he saw a value that was purely instrumental and technical, and opposed any efforts to strengthen or broaden its organisation. He had then come into direct conflict with Volterra, preventing him from absorbing the zoological station of Naples into the Thalassographic Committee, in order to defend the rights of the heirs of its founder, Anton Dohrn. In private, Volterra labelled Croce as a 'minister who was more German than Italian'.

Never say never. The idea of constituting a National Research Council definitely seemed on its way out when it was taken up again by the member of Parliament Paolo Bignami who, referring back to earlier projects, presented a proposal for legislation to create a central scientific-technical committee for the support of industry. In reality, Bignami's proposal differed from the findings of the Villa commission in several ways that were far from negligible. First of all, it centred the activities of the future committee on the financing (via grant mechanisms) of applied research, to the total exclusion of theoretical research. It also went back to the old scheme of encouraging the activities of inventors, which the organisations set up in other countries during the war had relegated to the sidelines. These are different accents and 'temptations' that crop up more than once in Italian policies regarding the sciences, even today. Bignami's proposal obtained the support of the committee in Parliament that was responsible for it, but failed without even being discussed when the Giolitti government fell. The political climate changed once more, this time in favour of Volterra's projects, with the Bonomi government that followed. This time the Minister for Public Education was Orso Mario Corbino, director of Rome's Institute of Physics where Volterra was a professor, a firm supporter of the need to support scientific research. Thus it was agreed to establish an extraordinary budget of ten million lire for the scientific laboratories. But once again there was no time to translate the steps taken by Corbino into law before the government fell again. With the end of the Bonomi government and the prolonging of the Italian political crisis in the Facta government that followed, the dominant concerns of the nation (as well as those of Volterra, as we will see) were far from scientific cooperation.

In 1922, with the advent of Fascism, the political-institutional context of Italy changed. This will be discussed in greater detail in the next chapter. For now let's finish up the story that led to the constitution of the National Research Council. On November 16, with the so-called 'Bivouac Speech', Mussolini obtained a vote of confidence in Parliament for his cabinet. As the first measure for the budget of 1923, the new Minister for Public Education, Giovanni Gentile, cut funding for university institutes by 30%. That event led to a new closing of the ranks between scientists and industrialists, so much so that in August 1923, Orso Mario Corbino was once again nominated Minister, this time of national economics. So in September, the funds cut by Gentile were reintroduced into the budget.

Up to then, political instability had not made it possible to solve the question that the scientific community had been grappling with since 1919, but the public had nevertheless in some way become aware of problem of the promotion and coordination of research. Now at this point it was the Fascist government whose attitude towards the problem was the most sensitive and efficient (!). There were at least two reasons that led Mussolini to put into action the proposals that his predecessors either wouldn't or couldn't turn into reality. First of all, the new prime minister wanted to broaden consensus by giving a sign of a return to political stability and renewed attention to technocratic motions brought forward in time by the nationalists. He was thus strongly in favour of recuperating the kind of government intervention used to mobilise national energies and resources during the war, and

that above all the Nitti and Giolitti governments had tried to erase. Mussolini, who was also ad interim minister for foreign affairs, was also sensitive to the subject of the cultural boycott of Germany, which as we have seen was the order of the day within the newly created institutions for international intellectual cooperation.

Thus, while as late as July 1923 the project to institute the National Research Council seemed to have been shelved, in August new signs of life appeared. We have already mentioned reasons for this. There was probably an intervention on the part of Corbino. There are also those unfathomable dynamics that characterise such a large part of Italian politics. Volterra wrote to Giovanni Magrini, who had collaborated with him as far back as the Thalassographic Committee: 'As far as the law of the International Research Council is concerned, to the letter from the Ministry of Finance who agreed to introduce a new item into the budget for foreign affairs, but presuming that the funds would come from cuts in other budgets, there has already been an energetic response to the effect that that is absolutely impossible and that a new allocation of funds is necessary. Given the tone of the letter, which was short and snappy, signed by Mussolini, I would like to hope that the [Minister of Finance] De Bellis will drop his opposition. Then it could go through as planned'. Just a few weeks later, the decree for the institution of the National Research Council was ready. Volterra wrote again to Magrini on 6 October: 'The agreement with the Minister of Public Education has worked well, the Accademia dei Lincei remains in charge of administrative management and the allotment of funds is part of the budget for foreign affairs. . . . I believe that nothing more could reasonably be expected'.

The decree for the institution of the National Research Council was effectively passed on 18 November 1923. The headquarters of the new agency was to be in Rome, in the Accademia dei Lincei. It aims were exactly in line with those of the international organisation of which it was part. Its structure was based on different national committees organised by discipline, which were members of their respective international unions (either already formed or in the process of being formed); the presidents and secretaries of these committees made up the Council, with the addition of one delegate from the Ministry for Foreign Affairs, one from the Ministry for Public Education, and one from the Accademia dei Lincei. Bonaldo Stringher, now governor of the Bank of Italy, was designated Administrator.

The National Research Council was thus strongly oriented towards international activities, while its supposed functions as an agency for the coordination and promotion of Italian research remained on the back burner and were actually quite limited in substance. The allotment of 175,000 lire was sufficient for carrying out the council's international responsibilities, but was absolutely inadequate for any possible national function. Just to give an example and means of comparison, the Thalassographic Committee alone had a budget of 300,000 lire! In compensation, the new council at least now existed, and its supporters could use its new status as an official institution as a lever for further developments. While the vagueness of its internal functions is a sign of the resistance encountered in the formulation of the decree, the fact that the Council took upon itself the task of explicitly defining its own role can be read as a demonstration of its capacity to formulate concrete

designs and plans. The by-laws of 1924 – the work of Volterra, Amedeo Giannini (a government advisor and the delegate of the Ministry of Foreign Affairs) and Stringher – recycled the essential elements of the documents of the Villa commission. It would also repeat the most controversial point – the establishment of national scientific laboratories – although in a way that was less categorical with respect to the 1919 project, which had seen the Institute for Aeronautics as the fundamental nucleus of the laboratories.

In January 1924, on the recommendation of the Accademia dei Lincei, Volterra was elected president of the National Research Council. The national committees that had been established at that time were those of astronomy, geodesy-geophysics, mathematics, scientific radio-telegraphy, chemistry, geography and physics. These represented the main innovation with respect to the project of 1919, and were conceived as links between different disciplines within Italy, as well as bridges to the same areas in the international arena. The structure created appeared to be potentially capable of achieving the ends that Volterra had already defined years earlier, while leaving the door open to future developments such as the national laboratories, which appeared to be beyond reach at the moment.

It was in this way that the Unione Matematica Italiana (Italian Mathematical Union) was created, as the mathematics committee of the National Research Council. Its first president was Salvatore Pincherle (1853–1936), but directing the operation once again was the long arm of Volterra. He wrote to Pincherle in March 1922: 'I have not forgotten the undertaking that you wished to entrust to me, to attempt to constitute the Unione Matematica Italiana, to be confederated with the International Mathematical Union'. Volterra's influence on the newly created professional association for mathematics is made clear in a letter written by Pincherle on 5 August 1924. The question concerned the selection of the 12 Italian delegates to the international congress to be held in Toronto. Pincherle reserved four places in the delegation for himself and for the office of the presidency of the Unione Matematica Italiana. A fifth place was naturally reserved for Volterra. Then Pincherle wrote, 'As far as the other seven are concerned, we ask you to be so kind as to name them'.

6.3   The *Encyclopedia Nazionale*

The constitution of the National Research Council was Volterra's most burdensome undertaking during the years following World War I, but not the only one. He carried on with his research, though to a lesser degree, and continued to travel in order to develop the international relationships that were not directly related to the International Research Council. In 1922 he was awarded an honorary degree from the University of Edinburgh; the following year he went back to Great Britain to take part in the Liverpool congress of the British Association for the Advancement of Science.

Even on the front of the reorganisation of scientific activity in Italy, the National Research Council was not Volterra's only enterprise. Immediately

after the war there was the project for an *Encyclopedia Nazionale*, conceived in 1919 by Ferdinando Martini and the historian Mario Menghini,[6] which Volterra supported and in which he found himself soon involved. The idea of creating an Italian national encyclopaedia had circulated for a number of years in the world of Italian publishing, at least as early as an attempt in 1907 by Treves, Barbera and De Marsico, three of the largest Italian publishers in the early years of the twentieth century. Martini and Menghini's project also involved the Italian Physical Society, and thus Volterra. His interest in it can be traced back to the years during the war, to the role he played in inter-ally intellectual cooperation, and to the relationships formed in that context even beyond the scientific sphere. The prospective of creating a national encyclopaedia, and the opportunity to make the voice and the needs of the scientific world heard, fit well into plans for a non-subversive modernisation of the liberal state supported by Volterra and many others working in different branches of Italian culture. As usual, Stringher was charged with overseeing the realisation of the project, taking care of the financial aspects. The undertaking required a hefty initial investment but sales of the first volumes were expected to make further progress economically feasible.

In January 1920 Volterra sent a first cost estimate to Stringher, but the experts he had consulted expressed opinions that were far from positive. The lengthy financial negotiations and the difficulties posed by Stringher began to try Menghini's patience. Volterra was the outlet for his complaints. However, in the end complaints and various other kinds of pressure must have had some kind of effect, because on 7 August Menghini wrote to Volterra, 'I feel it my duty to tell you that the fate of the enterprise appears to have taken a turn for the best. In fact, last Tuesday the Hon. Martini came here, and we went together to see Commendatore Stringher, who confirmed his full faith in the outcome of the work that was to be undertaken, and declared his approval of the formation of a syndicate that will for now provide the necessary funds for the preparatory work on the encyclopaedia, arranging it so the actual work of compilation begins when Italian industry can be seen to have taken a stable course'. The consortium promised by Stringher was formed during the early months of 1921, with the involvement of the Banca Italiana di Sconto, the Banca Commerciale Italiana and the financier Luigi Della Torre, co-founder with Giuseppe Pontremoli of a large publishing chain named "Società editoriale italiana". The publication of the volumes was entrusted to the Florentine publisher Roberto Bemporad, and Menghini was finally able to begin to contact the scholars who were supposed to contribute to the initiative.

In reality, the economic difficulties and the uncertainties regarding the banks remained, not least because Bemporad was in a precarious situation. The publishing house decide to entrust direction of the work to Giuseppe Fumagalli, one of the greatest bibliographers of the day and an esteemed organiser of editorial projects. However, Martini did not agree with this decision, nor did Menghini, who aspired

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[6]For the information that follows, see the exhibition catalogue *1925–1975. La Treccani compie 70 anni. Mostra storico-documentaria* (Rome: Istituto della Enciclopedia italiana, 1993).

to that position. In July 1922 Stringher wrote to Volterra, 'I hope that you won't fail to come, because we must not accept publisher Bemporad's decision, to which Hon. Martini is decidedly contrary'.

With Bemporad's proposal of Fumagalli rejected, Martini decided to accept Stringher's recommendation to entrust the execution of the encyclopaedia to a consortium formed especially for that purpose, which was to include several outstanding figures in the publishing world. The choice centred in particular on the Fondazione Leonardo, a foundation for Italian culture, and on Angelo Formiggini, a publisher in his own right who was also a founding member of the Fondazione. Martini had been the first president of the Fondazione Leonardo, and was now honorary president, while Ivanoe Bonomi was the current president and Orso Mario Cobino vice-president. Members of the board included Formiggini and various ministers, who nominated their own delegates: these included Amedeo Giannini representing the Ministry for Public Affairs, and Giovanni Gentile representing the Ministry for Public Education. Even before the war Formiggini and Gentile had been cultural adversaries: Formiggini accused Gentile of "doctrinal tyranny" and Gentile cordially shot back with accusations of dilettantism and eclecticism.

After brief but careful preparatory work, in November 1922 Bonomi signed an appeal to Italian editors that had been prepared by Formiggini but reviewed and modified, even substantially, by Volterra and Stringher. The circular was backed up by a series of meetings promoted by Formaggini and by a 'blue book' containing a preliminary outline and a first cost estimate for the encyclopaedia. The end of the following January the consortium of publishers and booksellers, known by its initials CIEL, was formed to oversee the realisation of the *Grande Enclicopedia Italiana*. However, once the economic problems were on their way to being solved, personal and cultural ones cropped up. Formiggini mentioned these in a letter to Fumagalli: 'The Italian Physical Society is a great hindrance, saying that because the thing is being done under their auspices, they want to be the ones to choose the authors. . . . I am quite worried about excellent Menghini, who came to complain and who considers the so-called idea as his own personal property, and says that is any case he wants to make a living off of it, which is a little exaggerated'. On the level of culture Croce too was rather critical. In this regard Formiggini wrote, 'the illustrious man told me that in order for the Encyclopaedia to turn out to be lively and vital it was necessary for it to have its own idea'. Menghini's criticism had the same tone. In a letter of 26 February to Martini, he wrote, 'the triumph of Formaggini's arrangement was that of Bemporad, and what we have here is no longer a scientific encyclopaedia but one like that of Larousse . . .. As soon as I can I am going to see Gentile[7] and I hope to interest the government in the enterprise'. On 21 February 1923, Formiggini was ousted following an assembly of the members of the Fondazione Leonardo, which he accused of being knowingly

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[7]In the meantime, Giovanni Gentile had become Minister of Education.

piloted by Gentile.[8] The vicissitudes of the foundation brought the project for the great encyclopaedia to a temporary standstill, although it was still seen as necessary. It was ultimately Gentile who re-proposed its creation, this time successfully, just a few months later.

The early 1920s were the years of Volterra's greatest visibility. The presidency of the National Research Council was flanked by that of the Accademia dei Lincei. In reality, in 1919 Volterra had also been elected president of the Società dei XL, the oldest association of scientists to call itself 'Italian'. However, he resigned that position in 1920 when he was elected vice-president of the Lincei, with Francesco D'Ovidio as president. Volterra was intensely involved in the Lincei during this time. The war had decreased the academy's activities, causing delays and problems in the periodical publications, which had suffered in terms of both the number of contributions as well as the frequency of issue. The experience of the war was also evident in the new political orientation. At the end of the war, the Lincei elected Armando Diaz, Vittorio Emanuele Orlando, Sidney Sonnino and Paolo Thaon di Revel to membership 'for patriotic merit'. Volterra's increased involvement coincided with the commitment of the Lincei to the creation of the International Research Council, which we discussed earlier. It also coincided with the launching of several broad-based publishing initiatives, such as the printing of the 54 volume *Atti delle assemblee costituzionali italiane*. Not long after, Volterra came to represent, in the eyes of a majority of its members, the man capable of making the Accademia dei Lincei the leading entity in the reorganisation of Italian culture. This resulted in the reforms in the by-laws passed in 1922, and in the increase in the academy's endowment.

In this battle as well Volterra could count on the support of Stringher who, though no longer Minister of the Treasury, was always at home – and always influential – in that department. In November 1920 Stringher wrote to Volterra, 'I spoke with the Comptroller General of State this morning about the Accademia dei Lincei. He is favourably disposed, and will pre-arrange things to as to obtain the approval of His Eminence [Filippo] Meda. He has moreover informed me that the Treasury must receive the relative proposal from the Ministry for Education, with which the Academy must deal officially. Regarding which I made some observations, for reasons that are well known to you.[9] Bureaucratically, the Minerva cannot be gotten around; but we can present to the Ministry of the Treasury a copy of the note sent to the Ministry of Education, so as to obtain a consensus for

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[8]Formiggini's accusations are contained in his anti-Gentile pamphlet entitled 'La Ficozza filosofica del Fascismo e la Marcia sulla Leonardo. Libro edificante e sollazzevole', which has become almost impossible to find today; the pamphlet's title refers to the lampoon that gave Gentile the nicknamed 'Fra Ficozza', from the word in Roman dialect that means 'a bump on the head'.

[9]The Minister of Education was Benedetto Croce, who was also a member of the Lincei but was opposed to the direction taken by the academy in that period, and moreover, was engaged in a dispute with Volterra.

the allotment of funds and be better able to proceed'. Later, after the fall of the Giolitti government, he wrote: 'Dear Senator, tomorrow I will see the new Minister of the Treasury and I will speak to him about the poor Lincei'.

Volterra also tried to have the Museo Copernicano housed in the Accademia dei Lincei. This museum possessed the most important collection for history and science in the capital. His hope was to make it the nucleus for that Museum of Science that had been talked about in Rome since 1883. The attempt brought no concrete results. Instead, he was more successful with the building up of the library, the establishment of new prizes, and the re-launching of publications. Volterra's election to the presidency of the Accademia dei Lincei in June 1923 was not only the achievement of one of his greatest ambitions, it was also a clear sign that the members shared his strategy.

Volterra was also the European referent for the area of physics and mathematics of the International Education Board, which was the international operative arm of Rockefeller Foundation. In the years following the end of the war, that group launched a program of international scholarships that would play a very significant role in promoting, among other things, the development of quantum mechanics in Europe and in the United States. The experiment for international scholarships was a development of that funded again by the Rockefeller Foundation for the National Research Council in the United States. Thus it is not surprising that Wickliffe Rose, a trustee of the Rockefeller Foundation, went to Rome in February 1924 to meet Volterra and invite him to collaborate with the foundation. In honour of Rose's visit, Volterra organised a private meeting in his own home, the Palazzo Fiano-Almagià, with some of the leading figures in the Italian scientific community. The first result was a scholarship that made it possible for Enrico Fermi to study in Leiden in the laboratory of Lorentz. Volterra introduced the young Italian physicist in this way: 'a young assistant of our Institute of Physics, Mr Enrico Fermi, wants to spend several months in Leiden to perfect his studies in Physics and Mathematical Physics'. For his part, Fermi, who had just arrived in Holland, felt the need to send news back: 'Illustrious professor, I arrived yesterday in Leiden, having had to delay my departure by some days because of an abscess that had formed near my ear, and which I had to have lanced. ... Again many thanks for the interest you have always shown in me, and sincere regards to you and your family'.

Thanks to Volterra's support, many Italian and foreign scholars were able to gain experience that was fundamental to their training in research centres and laboratories in countries other than their own: these included mathematicians such as Bruno di Finetti, Szolem Mandelbrojt (who, introduced to Volterra by Hadamard, spent a period of study in Rome), Robert Mazet and André Weil, and physicists such as Aldo Pontremoli, Enrico Fermi, Enrico Persico, Franco Rasetti and Emilio Segrè. Later Volterra also obtained funding from the International Education Board for the Bureau de Poids et Mésures and for the constitution of the Institut Poincarè in Paris.

This gives us a chance to mention the Bureau International de Poids et Mésures. To complete the list of duties and honours that involved Volterra during the years after the war, we must add that he also assumed the presidency of this agency,

succeeding Pietro Blaserna in that position. Along with the International Association of Geodesy, the Bureau was the oldest of the few institutions that existed before the first world war. It is logical that, given the climate after the war, that it would be find itself facing the need for reorganisation, and had to overcome numerous obstacles in order to become functional once more. Under Volterra's presidency, the Bureau came through a difficult moment of organisational and scientific transition brilliantly, with a revision of its institutional by-laws, the creation of new units of measurements in the fields of electricity and photometry, and the construction of new headquarters in Sèvres between 1929 and 1931.

Chapter 7
The End of Freedom

7.1 Opposition to the Gentile Reform

The planning and creation of Italy's National Research Council were carried out from the end of World War I to the early years of the 1920s. In order to follow the 3 years of Volterra's presidency we went all the way up to 1926. Now let's go back to a more orderly chain of events.

For both Italy and Volterra, history was turning a new, dramatic leaf. In 1922 the political crisis accelerated strongly. In 8 months, from February to October, there were two successive governments presided over by Prime Minister Luigi Facta: a record even for a country that is known for the short durations of its governments. 28 October was the day of the March on Rome: Mussolini militants known as *squadristi,* or Blackshirts, marched into the capital and King Vittorio Emanuele III – instead of signing the state of siege as requested by the prime minister – charged the Duce with constituting a new government, which officially took over on 31 October. On 16 November Mussolini made his first speech as prime minister, the so-called 'Bivouac Speech' (because of his boast 'I could have made a bivouac of this gloomy hall . . .'). His government would last, with various changes of ministers, until 25 July 1943. This was what would be emphatically referred to by its leading figures as the 'Fascist revolution'. The politics of the day were already characterised by violence and clashes, and many members of the liberal ruling class hoped that the Fascist movement, in becoming institutionalised, would help stabilise the political system, while at the same time cleanse it of its more obviously illegal activities.

Volterra worried about the political situation, and harboured no sympathy for Mussolini's movement. In a first phase, while he was opposed to the new cabinet, he was also resigned to the facts. Faithful to the monarchy, in his institutional duties Volterra decided on a line of general collaboration with the government that the king had wanted, aside from any personal opinion he might hold.

The first occasion for conflict arose with the package of reforms presented by Giovanni Gentile. A philosopher from Sicily, Gentile had been concerned with the

problem of schools and education for years, and during the Giolitti government of 1921–1922 had supported Benedetto Croce's attempts at reform. Gentile had analysed and contemplated the political defeat suffered by the 'old master', and now he believed that he had clear ideas about how to go forward.[1] On the other had, the desire to enact a reform that would carry out Croce's previous attempt brought him strong support from both pedagogists and those within the vast and influential number of Croce's followers in the academic world. The Casati Law, which had governed education since the time of Italian unification, had run its course, and reform of the system of schools and universities had led to only partial and contradictory results. Thus, even those who were not sympathetic to the Fascist movement, such as Gaetano Salvemini and Adolfo Omodeo, appreciated Gentile's attempt; above all they trusted in his capacity to see it through.

Volterra was of a different opinion: he could not ignore the fact that Gentile's actualism and Croce's idealism denied any cultural dignity to the sciences, and he soon began to express his dissent. When Gentile spoke of 'Masonic' resistance to the kind of reform that Croce promoted, he was certainly thinking of Volterra, among others. However, Volterra's diffidence was a minor fact. In general, Gentile's nomination as the minister of public education was met with a great outpouring of good faith, not only by the Fascists in education or by students who were politically close to the new government, but also by men who were hostile to the government but believed in Gentile as a person, and often also as a philosopher. The first move by the newly appointed minister was the repeal or block of a series of regulations that would have hindered his actions. This was on 14 November, just 2 weeks after the swearing-in of the new cabinet. It was followed by several interventions intended to reorganise central and local school administrations in order to reduce costs, while at the same time making the chain of command more effective: the number of general directors went from five to four, in such a way that Gentile could count on those he could trust absolutely; central accounting was restructured into a single division rather than the four previous ones; local administration was reorganised on a regional rather than a provincial basis, reducing the number of superintendents from 73 to 19. On 24 November the government was invested with full power (through all of 1923) to enact administrative reforms, thus providing Gentile with an exceptional instrument that exempted him from parliamentary control and placed him in a situation analogous to that in which the Casati Law was enacted. Gentile did not fail to understand the importance of having a consensus for his designs, and thus the key role that communication would play: to direct the ministry's press office he called Ferruccio Boffi, a classmate of his from Pisa and very aggressive member of the Italian and Fascist press.

The reform of lower and upper secondary schools was the foundation of Gentile's edifice. It was the first measure to be enacted, on 6 May 1923, probably at the request of Mussolini himself, who wanted to show the Vatican how solid his

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[1] On Gentile and the reform that carries his name, see Gabriele Turi, *Giovanni Gentile. Un biografia* (Florence: Giunti, 1995).

government was even after the removal of public ministers. On the other hand, these schools had always been at the centre of Gentile's attention. He had two fundamental concerns: to put into effect a strict system that established who was permitted to enter university, and to make the humanities the basis for the education of the ruling class, and therefore to give them a privileged place in the secondary school curriculum. This led to the abolition of the 'additional classes' (the extra year of classes that made it possible for those who had attended certain non-accredited kinds of schools to go on to university) and the institution of two kinds of courses that were terminal. One was the so-called '*complementary schools*' (commonly known as 'work preparation'), open to the 'modest citizen' after primary school, and the 'minor professions'; this took the place of the technical schools, which had previously provided access to more advanced studies, an avenue that was now closed. The second was the *female lyceum* for the 'young girls of the bourgeois', conceived in keeping with the subordinate position of women inherent in Gentile's idea of society. Also foreseen were a sector for artistic training and a new kind of technical training that would last 8 years, following primary school, which did not permit access to university and absorbed some of the courses of the earlier technical institutes. The physical–mathematical course of study was instead transformed into the scientific upper secondary school (*liceo scientifico*), which lasted 4 years, which was entered after the lower secondary school (*ginnasio*), and did not provide access to the faculties of humanities at university. For the training of teachers for elementary and middle schools, the earlier *scuola normale*, or normal school, was replaced by teacher training schools lasting 7 years, entered directly after primary school, the aim of which was qualification.

Only one kind of program permitted entrance to all university faculties: primary school followed by 5 years of lower secondary school (*ginnasio*) and 3 years of upper secondary school with an emphasis on the humanities, including Latin and Greek (*liceo classico*). This was the school for culture *par excellence*, the one that trained the ruling class. But the humanities permeated secondary schools overall: Latin was required in all 3 years of lower secondary schools (except for the complementary schools), as it was in all lines of study in the upper secondary schools. Further, the lower secondary school, that is, the first 3 years of classical secondary school, provided access to all the other lines of study of upper secondary schools. Scientific courses were further penalised by several groupings of subjects: the chairs for mathematics and physics were combined and the overall number of hours for both subjects reduced. The same was true of history and philosophy, but with a combined numbers of hours that was greater thanks to an increase in the teaching of philosophy. Further, the history of art was made obligatory. In the educational designs of the secondary schools, a distinction was made between the function of lines of study aimed at professional training, destined for the more modest social classes, and that of the 'aristocratic' and 'disinterested' ones foreseen for the ruling classes. Gentile, who had been beating this drum since 1902, clarified his intentions in an interview of 29 March: 'I intend to concentrate the function of the secondary school in the classical school: which due to its value for the nation and for education will have a clear pre-eminence over other schools destined for the formation of the students' minds'.

In any case, the reform had to take into account the financial limits imposed by Minister of the Treasury Alberto De Stefani: the increase of teachers' salary was compensated for by a reduction in their number, obtained by retiring a part of them, by combining of chairs in secondary schools, and by prohibiting women from teaching subjects considered to be 'typically masculine', such as philosophy, history, law and political economics, in upper secondary schools and technical institutes.

All of these interventions reflected of the idea of a state that is strong, hierarchical and authoritarian – an idea held by Gentile and by the government as a whole– which set the tone for all administrative reforms. The reforms of schools was also supported, for at least the first 4 years, by the Catholic sphere, thanks to the different status granted to private schools and to religious instruction in primary schools being made obligatory. The possibility of a dialogue with the Catholics, thanks to the acceptance of some of the requests of the ecclesiastical hierarchy that had been unheeded up to that point, was one of the elements that guaranteed Mussolini's total support of Gentile's proposals. On the other hand, Gentile did not consider these to be merely instrumental aspects: to the contrary, they were part of a design that aimed at imprinting the education received in schools with a strongly hierarchical character, with lines of study that were differentiated in such a way as to restrict social mobility.

The reform of secondary schools aroused numerous negative reactions. Among the mathematicians, it was the Mathesis Association that expressed the first strong doubts and dissatisfactions. The protests of the association tended to concentrate on the combined teaching of mathematics and physics in the upper secondary schools, in a number of class hours that was sometimes less than those previously dedicated to mathematics alone. The large majority of teachers came out against such combined courses. There were various reasons: the different qualifications required for teaching the two subjects; the difficulties of putting the reform into effect with a body of teachers that was unprepared; the intolerable teaching load, even the 'constitutional' differences between mathematics and physics. University professors too were on the verge of revolt, struggling with rampant rumours of all kinds about the minister's intentions for the expected university reforms. The professors who had seats in parliament held a series of meetings – the first of which in April – with the aim of making their voices heard and give weight to their own point of view, in spite of the advantage that the grant of full power had given Gentile. In the meeting of 25 May, the topic of secondary schools was raised by the Italianist Guido Mazzoni, who, relating the reform of secondary schools with the forthcoming reform of universities, proposed a motion requesting that the measures be suspended. Mazzoni was joined by Volterra, who 'criticised the minister's plan, which diminishes and abases scientific culture in secondary schools. The fusion of mathematics and physics finds the professors unprepared to assume dual instruction. On the other hand, the dispositions natural to one and the other science are different. A good professor of mathematics cannot have the attitude towards experiment that is necessary for a professor of physics'. Even Antonio Garbasso, a Florentine physicist who was a sympathiser with Mussolini and was the first Fascist official in Florence, was opposed to the reform. He wrote to Volterra

on 6 July 1923: 'Here we are all dismayed over the "reform" of the Hon. Gentile. Without mentioning the interests of science, we ask how it is supposed to be possible, should the necessity arise, to obtain from the young philosophers of the future those thousands of sublieutenants of the corps of engineers and the artillery that had to be recruited in a few months during the last war. Is it not possible to illuminate Hon. Mussolini?'

The reorganisation of secondary schools was followed by the reform of the High Council for Public Education. This council had originally been assigned responsibility for primary and secondary schools by the Casati Law, but this had been taken away by later legislature. Now these responsibilities were restored, and the number of members reduced to 21, all of whom were nominated by the king under the advisement of the minister for public education. This meant a return to the Casati Law and to the times before the reform of 1881, which had introduced membership by election. It was the first time in decades that the principal of election for agencies of self-government had been abolished.

With the support of several members, including the mathematician Guido Castelnuovo, and obviously Mazzoni and Volterra, the Academia dei Lincei decided to address the problem in its June session. Volterra was aware of how central the reform of secondary schools was to the new political direction taken, and that it was necessary at the very least to postpone its enactment. It was clear to him that the opinions of the academic world would not be taken into consideration at all by Gentile, unless public opinion could be swayed into agreement with them. It was thus necessary to try to block the reform of secondary schools and try to act early on the reform of primary schools and universities. It was also urgent. Volterra wrote to Garbasso on 15 July: 'in the last session of the Accademia (Class of Moral Sciences) a motion was proposed and approved to form a commission to assess the reforms. I would have liked to have added your name, but given the urgency, and not being able to count on your coming to Rome, I wasn't able to do it. After a lengthy discussion, speaker Castelnuovo compiled a report that was, in my opinion, very well done, which serves as the commission's statement'.[2] This report was sent to all academy members on 15 July. This meant that Gentile received a copy too, and thus read: 'thanks to enlightened legislators Italian schools between 1860 and 1880 had risen to a high level and could compete with the best schools abroad. Deplorable leniencies and laxity of discipline had perhaps in recent decades diminished the effectiveness of schools; but had the most austere rules remained in vigour, a firm hand would have been sufficient to restore to schools the prestige they once had, even while taking into account the new demands brought about by our nation's cultural and economic progress. A radical reform, even though inspired by noble intentions, did not appear to be necessary'. The position taken by the Accademia dei Lincei seems to have been inspired by a defence of the status

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[2] The Commission was largely composed of members who were also senators and who had participated in the meetings in April and May. For its exact composition, see Giovanni Paoloni (ed), *Vito Volterra e il suo tempo*, op. cit.

quo: Italian schools, especially after the war, showed signs of weakness, but these could be remedied by returning to the 'backbone' and rigour that had made the post-Unification achievements possible. In short, it was the Fascists who wanted change – even if those changes were aimed at the restoration of earlier times – with all the advantages that derived from proposals presented as new and capable of remedying the backwardness experienced up to then.

The Lincei's report made it to the newspapers, with an article by Nicola Festa (a member of Azione Cattolica, a widespread Roman Catholic organisation) in the *Giornale d'Italia*, and an interview with Gentile himself that appeared on 18 August in *La Sera*. Gentile said that 'the particular criticisms raised by the report of the Accademia dei Lincei are not such to make it advisable for me to change the reform ... If I were of a mind to use an *ad hominem* argument, I would begin by questioning my critics capacity to criticise me. All of them are illustrious scientists, to be sure: but how many of them have made issues related to schools the substance of their intellectual work as I can boast of having done? This is why the criticisms raised are quite weak and inconsistent, and in any case it is unlikely that that report has unanimity, or even a simple majority within the entire scientific world'. Gentile knew quite well that the university community was divided, and that it was unlikely that those of its members who were against the reform would come out into the open. 'Everyone is alarmed by the Gentile reform', wrote Volterra to Garbasso. 'Our Faculty has voted on a motion, but not one as vigorous as I might have wished. The proposal to send it to the Prime Minister didn't pass either. I was one of the few who voted in favour of that'. A difference began to be seen among members of the Lincei as well. Castelnuovo wrote to Volterra, 'When you return, we will have a lot to discuss. In the first place the letter by D'Ovidio,[3] so very inappropriate, because, using a detail of the report as a pretext ... it takes advantage of the opportunity to show the public that he is not at all responsible for the report, and that the sinner is the new president!'

Already in the May and June sessions, those senators who were professors and Catholic had dissociated themselves by voting against Mazzone's motion. In addition to support by the Catholics, the reform could count on the many followers of Croce and Gentile working in secondary schools and universities. On 21 August, a new article in favour of the reform appeared in the newspaper *Il Popolo* run by Don Luigi Sturzo. Castelnuovo wrote to Volterra: 'Dear Volterra, 2 days ago I sent to the Giornale d'Italia an article replying to the Minister and to Prof. Festa. I treated the Minister with a great deal of deference, because he is the Minister and because, his extravagant ideas notwithstanding, he is a person of stature, although one who wrongly believes that all critics are motivated by personal interests. The inconclusive article by Prof. Festa didn't actually merit being taken seriously, but I tried to avoid controversy, and I have repeated those two or three points on which we were in agreement. Again with regard to our report, I only need

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[3] This was Francesco D'Ovidio, Volterra's predecessor as president of the Accademia.

to tell you about an article in the Popolo (that must be the newspaper of Don Sturzo) on 21–22 August, in which, after having rebuked the Accademia for being too removed from the life of the nation, and after having said many things that weren't exactly correct, it finds that this first attempt [i.e., the report] at taking an interest in national culture was an unsuccessful one. Whoever wrote it (the article is unsigned) is obviously a follower of Croce or Gentile, and even criticises Garbasso'. On 15 September a press release issued as usual at the end of a cabinet session, expressed the government's full support of Gentile: 'Worries and accusations were cunningly spread from place to place in academic circles, where the opposing camp was believed to have had some followers . . . who did not yet want to concede recognition of the fact that the cultural rebirth of the country is one of the primary points of the government presided over by the Hon. Mussolini'. The overstatement of consensus was preparatory to the issue of the directive regarding university reform. On 2 October Castelnuovo wrote again to Volterra: 'The people with whom I have spoken are unanimous in criticising the Gentile reform, but very few of them will have the courage to do so openly. We will discuss the tactic to use in person when you and Scialoja have returned'.

Like that of secondary schools, the reform of universities, already preceded in April by nervous rumours that some universities were going to be abolished (an intention that Gentile always denied), rested on the treasury's need to justify an increase in taxes and cuts in the budget.[4] The reform aimed at reducing the number of students, who would otherwise – according to Minister Gentile – have only lengthened the queue of the unemployed. In reality, the dynamics of enrolment was not substantially changed, but Gentile's measures nevertheless radically altered the physiognomy of Italian universities. With the processes of election revoked, school principals and rectors went back to being nominated by the ministry. The introduction of post-graduate examinations for professional qualification reaffirmed the purely theoretical and scientific nature of university studies, according to a conviction long held by the minister. Finally, liberty and autonomy were understood simply as a lightening of the burden on state finances: a distinction was made between the ten largest universities, which were totally government funded, and the smaller ones whose fate depended on agreements between the government and other entities. Finally, open universities were recognised, over which the government reserved the right to check if they were qualified to grant degrees. Gentile called the Franciscan father Agostino Gemelli to collaborate in this part of the reform. This was the premise for official recognition of the Catholic University, which took place in October 1924. Finally, on 1 October, reform of primary schools was also enacted: 'the foundation and highest aim of all levels of primary education reside in the teaching of the Christian doctrine according to the form given it by the Catholic tradition'. Religious instruction was assigned to teachers or others deemed qualified by ecclesiastical authorities, while parents retained the right to ask that

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[4] The budget cuts were only partly compensated for by a special allocation of funds for university laboratories proposed by Corbino, who was the new minister for the national economy.

their children be exempted from such teaching. The other programs as well had to carry the imprint of religious tenets. In a circular of January 1924, the minister wrote that 'divine religion is a depiction of the absolute that reveals immediately to the child his duties as a man' and that 'civilisation is synonymous with Christianity; that no faith conceives the divine in such a human way'. Earlier Gentile had clearly indicated – in an interview given on 2 June – the only possible alternative for teachers who were either not Catholic or did not believe in God: 'if someone does not want to be a schoolteacher he can always do something else, and above all he can think what he wants, but schools, being Italian, and therefore Catholic, must see to the needs of the Italian people'.

7.2   Watched by the Police

1924 was the year that the crisis faced by the constitutional system that had accompanied the development of Italy following unification came to a head. Volterra was busy above all with the organisation of the National Research Council, of which he was elected president during the plenary session of 12 January. The political defeat in the battle against the reform of the schools and universities was in some way compensated for by the founding of the National Research Council, with which Volterra obtained a long-lasting result which echoed for years to come. He could thus hope to continue to work for Italian science through the Council, in spite of the Gentile reform. The reform itself was met with a large number of negative reactions. Among these were those of the Catholics, who had hoped for ulterior concessions regarding both the extension of obligatory religious studies to secondary schools as well as a positive answer to their requests for special considerations for Catholic private schools. Criticism regarding the disqualification of technical education came from the sector of industry as well, which was seriously concerned about the ramifications for industry of the radical reform of secondary schools. The criticisms expressed gave some ray of hope, which was increased by the obvious weakening of Gentile's position, who had already by February announced his intention to resign (he actually did resign in June).

The elections in April 1924 took place in a climate of the violent intimidation of the opposition, with serious infringements on freedom of expression due to the heavy use of government censure of the press, and an electoral law that guaranteed the award of absolute majority to the party that was expected to obtain a relative majority, that is, the Fascist party. There were also numerous cases of fraud and cheating. These were protested against by the Socialist party, which contested the outcome of the election. The party secretary, Giacomo Matteotti, was abducted in the early afternoon of 10 June, just as he was leaving home to go to Parliament. Murdered immediately after, his body was only found many weeks later. The jumble of rumours that began to circulate in the space of just a few hours after the kidnapping were an attempt to mislead the public, but did not succeed in concealing the fact that Mussolini was the political instigator of the abduction.

What was less clear, and remains so even at a distance of many decades, are the contents of the planned speech and the documents that Matteotti carried with him, since these were never found. The hypothesis advanced at the time was that he was to have delivered material to Parliament that would have nullified the results of the election beyond the shadow of a doubt. Other hypotheses spoke of proof of a case of corruption in which Mussolini was involved. In any case, the public saw the assassination of Matteotti as the latest in a series of violent episodes aimed at intimidating the opposition. This was the common denominator of the indignant reactions that seemed, for the first time since 1922, to shake the political foundations of the Fascist government. The Matteotti affair convinced those opposed to Fascism that the usual weapons used in political battles were by that time inadequate for taking on the threat posed by Mussolini's politics. With the exception of the Communists, led by Antonio Gramsci, the opposition parties (already penalised by the award of majority to the Fascists) decided to delegitimise the legislature and the government, abandoning it in protest. The Parliament was on Rome's Aventine Hill, the event became known as the Aventine Secession, and the members of Parliament who left were known as *aventiniani*.

Gentile resigned on 14 June. For some time his friends and former students had been urging him to separate his own destiny from that of the powers of Mussolini, but that was not the main reason behind his gesture. He stepped down so that the discontent raised by his measures would not further damage a government already in difficulty. Convinced that Fascism was the best guarantee for 'his' reform of schools, his resignation was designed to protect his creation. Mussolini wanted to call Croce to take his place: a clever political move foreseen by Croce, who had made his unwillingness known to Gentile and thus opened way for the nomination of Alessandro Casati, vice-president of the High Council for education and a liberal member quite acceptable to Gentile. Mussolini accepted the proposal, and named Casati, with Balbino Giuliano flanking him as undersecretary. Giuliano was extremely loyal to the Fascists and to Gentile. In one of the first public appearances (an interview with Giuseppe Prezzolini, who had been asked by Gentile to do it), Casati made the reform his own, even though he declared himself to be in favour of some of the modifications requested by liberal and Catholic members of parliament. At the end of August, after Matteotti's body was discovered, Casati defined the Aventine secession as 'either completely in bad faith or childishly silly', declaring that he was convinced that 'we must not turn back if there is truly to be an Italy in the world'. So, while the Aventine secession didn't have an political outcome hoped for, it did prompt Mussolini to constitute a commission to reform the government, the so-called 'Commission of Fifteen', calling Gentile to preside over it. It was this commission that established the bases for the measures that, in the months to follow, would make it possible for Mussolini to transform the constitutional monarchy and parliamentary government into a dictatorship in the proper sense of the term.

In the second half of 1924 Volterra followed the political situation with increasing alarm. It was by this time clear that Fascism represented the end of the liberal state he had always believed in. He was not alone in thinking this. The break

between Croce and Gentile in October sounded like a farewell. In October, after the congress of the Italian Liberal Party had relegated those who supported collaboration with the Fascist government to a minority, Gentile used a phrase that Casati had reported to him as a pretext for writing to Croce to ask for a clarification of their relationship, which had up to that time been one of 'reciprocal esteem and warmth'. 'In truth I don't have any recollection of having said those things to Casati', Croce replied, 'and I could not have said them because they don't correspond to my feelings. To be sure, for many years we have found ourselves in a mental disagreement, but not one in any case such that it would reflect on our personal relations. But now this has been joined by another of a practical and political nature, and indeed the first has been converted into the second; and this is more bitter. Nothing can be done. The logic of the situation has to unfold through individuals and in spite of individuals'. For Croce, the rupture with Gentile was a painful but inevitable personal passage in the definitive separation from Fascism. It was the break-up of a friendship that had lasted for 30 years. The two philosophers no longer even acknowledge each other when their paths crossed in the Senate.

On 31 October 1924, Volterra received a letter announcing that 'on 8 November there will take place here in Rome a gathering of members of Parliament, scholars, combatants, to assemble the free men of constitutional opposition in a "national union of liberal and democratic forces". ... It is our desire that, at the moment in which the Union is announced to the nation, that your authoritative adhesion, illustrious senator, will not be lacking'. A postscript informed him that many senators, politicians and writers had already adhered to the initiative, which was promoted by Giovanni Amendola, among others. Volterra responded to him by the next post in a telegram: 'Painful circumstances prevent me from participating session wish to express my profound sympathy national union liberal democratic forces'.

Let's go forward with the events of those months. On 3 January 1925 Mussolini, addressing the Parliament, openly assumed political responsibility for what had happened, including the death of Matteotti, challenging the weakness of the opposition. His speech, and the measures that followed it, are generally considered to be the actual beginning of the dictatorship. On 4 January, Gentile wrote to him to congratulate him on his 'excellent' speech: 'the whole nation awakens and returns to you'. While the violent wing of Fascism (led by Farinacci) unleashed a new wave of attacks, Mussolini was aware of the need to create to a more focused collective consensus, appealing to those intellectuals who by that time represented the only source of a possible threat to the dictatorship being constructed, given the absence of organised political and legal alternatives. He thus made Gentile responsible for reinforcing Fascism where it appeared to be weakest: the world of culture. As late as March 1925, Giovanni Amendola's newspaper *Il Mondo* asserted that it could register the failure of Fascism's attempt to 'draw into its orbit men of study and doctrine, to surround itself with the so-called intellectual class'. It was in this context that Gentile accepted the task of reorganising Italian culture in a way that was instrumental to the dawning regime, transforming of his philosophy into ideology and his followers in positions of academic power into functionaries of

the cultural agencies being constructed by the party and by the government. Croce's judgment was harsh: 'mixing politics and literature, politics and science, is an error which, when committed, as in this case, to support deplorable acts of violence and arrogance and the suppression of freedom of the press, cannot even call itself generous'.

On 29 and 30 March Gentile attended the Congress for Fascist Culture in Bologna, organised by the press and propaganda office of the National Fascist Party and the Fascist university, which Gentile himself had inaugurated on 9 March in that city. The key point of Fascist propaganda was, and would always be, the identification of the Fascist party with Italy: 'Our party is neither a sect nor a clique. Our party wants to be the Italian people'. From this followed the need to respect 'the old Italy, which we cannot erase'. To the extreme fringe of the Fascist party Gentile declared that the objective to be pursued was not that of devising a Fascist doctrine that was rigidly defined, but rather to promote the 'vast idealist contents' of Fascism to achieve a 'Fascism of culture'. At the end of the congress, motion was passed to invite Gentile to prepare a document that would make the characteristics of Fascism known in Italy and abroad. Thus was created the 'Manifesto of Fascist Intellectuals', compiled by Gentile and 'corrected by Mussolini's own hand', made public on 21 April, the anniversary of the founding of Rome.

It is clear that was being created was a vast, systematic 'program for the Fascistisation of culture and education'. Amendola then organised a response to Gentile's Manifesto, charging Croce with drafting it, which said: 'the Fascist intellectuals, gathering at a congress in Bologna, have addressed a manifesto to the intellectuals of all nations to explain and defend the politics of the Fascist party to them. In preparing to do so, those eager gentlemen must not have recalled a similar, famous manifesto that was proclaimed to the intellectual world at the beginning of the European war by German intellectuals; a manifesto that was met at the time with universal reprobation, and was later considered an error by the Germans themselves'. The counter-manifesto (today known as the Manifesto of the Anti-Fascist Intellectuals, or the Croce Manifesto) was published on 1 May in the opposition newspaper *Il Mondo*. Lengthy lists of supporters were published. Volterra's name appeared in the list published on 10 May.

A month later, Volterra carried out his final duties as president of the Accademia dei Lincei and the National Research Council. The sessions of general assembly and the meetings of the board of directors were filled with bitter disputes on the possible establishment of a large national laboratory, promoted by Volterra and Paternò, as well as by the government via Giannini and Magrini. The initiative was unwelcome to the majority of the academic community, which preferred to see an increase in funding for university laboratories. The tension created had repercussions in the Senate as well, and Giannini and Magrini became convinced that Volterra – who had already been identified as an opponent of the Fascist regime – was no longer of any use, not even on a strictly academic plane. There were other political disillusionments in store for Volterra. To his colleague Mario Abbiate, who had invited him in August to take part in a group of democratic senators being assembled, Volterra had the good sense to reply, 'I ask myself whether, to strengthen the

opposition, it is not better to form one single group, that is, one that includes the liberal group, even more so since the objectives at this time are shared and it is best that as far as possible the efforts are linked and perfectly in agreement'. It is easy to imagine his disappointment at Abbiate's reply: 'the constitution of different groups according to different political ideas is a fundamental necessity of the Parliamentary government'. In Volterra's way of thinking, the incapacity to aggregate liberal senators (who already constituted a Parliamentary group) and democratic senators further weakened the only possibility for legal opposition that remained.

In January 1926, Mussolini began to promote the creation of a new national academy, the Accademia d'Italia. Its founding in fact signalled the delegitimisation of the Accademia dei Lincei. Volterra, partly as a result of pressure from several members of the Lincei, prepared a draft of a letter of resignation: 'I feel that in this moment my person at the direction of the Accademia cannot be of use to it, and this opinion has also been made known to me by several members. While I thank my colleagues for the faith that they have always demonstrated in me and to whom I have attempted to reciprocate in the best possible fashion I could, I beg them to accept my resignation of the presidency'. News of the presumed resignation appeared in the press, with the result that several members immediately proposed a motion to ask Volterra to reconsider. A few days later Luigi Errera wrote him: 'By now anyone who thinks for himself and does not feel like making a show of submission to the ruling powers cannot remain in prominent positions. I believe that there will be more than a few in Italy who are sorry as I am, and considering the elevated position that you hold in science and in the esteem of the world, the news of your resignation will make a pitiful impression abroad, and will certainly not confer prestige on our nation'.[5] The news carried in the papers was not exact, but in June, when his 3-year mandate as president ended, Volterra decided not to run for a second term, and Scialoja was elected in his place.

In the meantime, the stage was being set for his exclusion from the National Research Council as well. In January 1927, responding to Somigliana's request for news, Volterra wrote, 'I too have seen Magrini only rarely. He told me too that new measures were in preparation for the Research Council, but I have never learned what they are, except that there is supposed to be an increase in funding. As for me, my term as president finished at the end of last year when the 3 years were up, and another president had to be nominated. He is the one who has to deal with the questions pending'. In fact, at the end of March the National Research Council was reorganised by decree, without, however, that new organisation having any practical effect, due to the absence of a president and a precise financial allocation. In August 1927, rumours were still circulating;

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[5] According to Giorgio Boatti, *Preferirei di no. Le storie dei dodici professori che si opposero a Mussoli*ni (Torino: Einaudi, 2001), Luigi Errera was a 'high official in the railroad who seeing Fascism grow decided to retire in order not to have to join the party, and retired right away, a month before he was to be promoted, so as not to gamble on his pension'.

Somigliana wrote to Volterra: 'I haven't had any direct information or communication. I heard that the contribution for the National Research Council was raised to 500,000 lire and that Marconi had been designated as president. Nor could Gen. Vacchelli tell me anything when I asked him. Magrini says that he was the one who obtained the increase in funding and who had been consulted on the new constitution of the Board'. What Somigliana had heard through the grapevine was true: in September Marconi took over as president, and the new board began to redefine the Council.

'My dearest friend,' Volterra wrote to Stringher in April 1928, 'I am writing you in confidence to tell you something that surprised me greatly, which was completely unexpected by myself and even by the others who were present here'. Stringher, who was still vice-president of the International Research Council, had been in Brussels to take part in the plenary meeting of the Council. 'The International Research Council has renewed all of its positions, both of those who were present, whether they were delegates or not, and those who had not even intervened from here. I intervened from here as I told you, before your departure, in the quality of international vice-president, a position that has no connection to either Italian institutions or with delegations. Now the renewal of my appointment, which had been put with all the others, has been opposed by the two Italian delegates, just as they opposed my involvement in a commission to study the regulations. ... The delegates are Giannini and Magrini. It seems to me that the Research Council should send delegates who have a bit more scientific credit and tact'. What Volterra did not know was that this step had been decided on during a meeting of the Board of Directors of the National Research Council, a meeting in which Stringher had also taken part; he also didn't know that the government had taken steps to exclude him from holding any international position. The conclusion of the letter to Stringher is filled with a great sadness: 'I wanted to tell you about this in strictest confidence, because even though I am no longer part of the National Research Council, which I founded with your help after much effort, I still have a great interest in the institution'.

This episode makes it possible to see the degree of institutional isolation that the regime forced on one who was an illustrious academic but a political opponent. Naturally, for Volterra there remained teaching. Here is a recollection of a somewhat special student, the future winner of the Nobel Prize for physics, Emilio Segrè: 'As a required third course I attended mathematical physics taught by Volterra. I add that the following year I attended his course again because he changed the topic each year. Volterra's lessons were well-organised and the material intelligently chosen (as I later became aware), but they were given in a voice that was soft and a little high-pitched or nasal, which tended to make one sleepy. ... Volterra had the habit of closing his eyes when speaking or calculating, and someone said that, being a man of good heart, he didn't want to see how much his students suffered. Aside from this, the lessons were instructive. Methods

of classic mathematical physics like Green's functions and the secrets of the Laplace operator were learned there.'[6]

By 1928 the police were keeping a personal dossier on Volterra, as a person who was politically suspect. As a senator, however, he was protected by statutory immunity, and sought relief from the atmosphere of stagnation and persecution in frequent trips abroad. He went to Sweden and Paris, where he held a series of lectures at the Institut Poincaré. He also went to Romania to visit the universities of Bucharest and Cluj, and to Madrid.[7] In 1926 received another honorary degree, this time from Oxford University. However, his passport was a national affair. On 19 December 1928, the head of police in Rome wrote to the General Direction for Public Safety, 'The Senator of the Kingdom Vito Volterra has presented a request for the concession of a passport to foreign countries. Same, as is known, is a liberal opponent, and finally, voted against the law regarding the Great Fascist Council. However, this office does not have specific reasons for denying his request. In any case I inform this Hon. Ministry regarding the decision that I believe should be adopted in this regard'. The request is lined with notes by functionaries responsible for researching the request: 'He was among the signers of the Croce Manifesto'; 'He is one of the opposing senators'. Other notes indicate that his name appears in police dossiers begun in 1926 and 1927 in categories C2 (subversive movements) and A1 (information on people). The 'Volterra case' rose to the upper echelons: the 'sì' given to his request for a passport is initialled with the characteristic 'M' that marked the matters directly acted on by the Duce. Matters regarding Volterra went all the way to the top, and he was the object of specific discussions between Mussolini and the chief of police Arturo Bocchini. Volterra would remain under police surveillance all the way through the 1930s, and his movements recorded in reports kept in police headquarters. When he returned home from his trips the functionary duly recorded: 'Inform that senator Vito Volterra son of Abramo has returned here. Reactivate surveillance'.

7.3 Swearing Allegiance

Article 18 of the university reform of September 1923, in addition to abolishing the criteria of election for the nomination of rectors and deans, made it obligatory for university professors to swear the oath of allegiance already required for public officials. The precise formula of the oath was, however, left up to a later regulation. Gentile, inaugurating the session of the High Council for Public Education on 15 November, with a complicated speech that would be met with mostly positive

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[6] Emilio Segrè, *Autobiografia di un fisico* (Bologna: Il Mulino, 1995).

[7] The text of the lessons was published in Spanish in 1927 and for an international readership in 1959 with the title *Theory of Functionals and of Integral and Integro-Differential Equations* (Mineola, NY: Dover Publications) with a foreword by Griffith Evans.

reactions, had specifically addressed this matter, referring to the non-neutrality of the ethical state, which 'has a faith, a doctrine to defend'. The victory in war had led to 'the highest, the most Italian concept of liberty, which is value, selection, hierarchy, that unite the State and the citizen into a single consciousness and a single will'. But the overall feelings doesn't appear to have been very favourable to what was perceived as a restriction of the freedom of professions, and the formula that was approved turned out to be rather banal: 'exercise the office of teacher and fulfil all academic duties with the aim of forming citizens who are industrious, upright and devoted to the Nation'. Further, the obligation regarded only those professors who were tenured or transferred to universities that were either completely or partly state funded. In spite of this, approval was granted with just 11 votes to 9.

On 13 January 1927 a new decree, emanated in the context of 'extremely Fascist laws' introduced into the oath the obligation of university professors to abjure 'associations or parties whose activities are irreconcilable with the duties of their office', providing for the exclusion from competitive examinations and the right to teach for political reasons. In defending the legislation before the Senate, Gentile declared that the new measures did not 'harm the principal of freedom of teaching, except for those who are still tied to the old prejudice of a neutral and agnostic State'. In January 1929, Gentile returned once again to the question of the oath, suggesting new revisions in order to 'resolve the delicate and by now urgent question of the fascistisation of Italian universities'. In February, the mathematician Francesco Severi, one of the signers of the Croce Manifesto, who later converted to Fascism and was 'rewarded' with a nomination to the Accademia d'Italia, asked Gentile to present a new formula to the Great Council: 'it is necessary that the measure be represented as a strict requirement aimed at achieving the much requested fascistisation of the universities: as an appeal to the loyalty of the professors, who cannot fail to take the oath without incurring consequences that were much more serious than being retired from their positions'. But at the same time, as an amnesty for political acts committed long ago by now, so that the State, in the technical sphere, can avail itself of every professor who has taken the oath; thus eliminating the absurd present situation of many professors who are only partially made use of, since they cannot even take part in evaluation commissions'.[8] (This was, in fact, Severi's own situation.) Gentile accepted the suggestion in a text of August 1929, in which he spoke of the still present phantom of the signers of the Croce Manifesto, and invoked the need to 'eradicate the memory of a past that is far behind': 'I have heard of a formula for the oath in which teachers would be called upon to swear allegiance to the regime as well: a step of that kind would demonstrate the alignment of all the instructors, with very few exceptions'.

Gentile's hope soon became a reality. Balbino Giuliano, who had become minister of national education, passed the law of 28 August 1931 that obligated all tenured and contracted professors to swear an oath of allegiance with a new

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[8] For more details, see Angelo Guerraggio and Pietro Nastasi, *Matematica in camicia nera*, op. cit.

formula that explicitly required loyalty to the regime. If there are any doubts as to Gentile's authorship of the measure, these can be laid to rest by the testimony of Gaetano De Sanctis: 'Gentile went on, saying that he himself had devised the oath as a means of invalidating the Croce Manifesto. "You know well that among the signers are many who have already retracted their signature. The oath offers them a way to withdraw it without an explicit retraction". "But I", I replied, "when I sign my name I do so for sound reason and never retract it". Gentile said that my decision pained him, told me that he aimed, by means of the oath, to liberate the universities from political intriguers and mentioned several names, but said that he never expected that I would be expelled from the university. Here I have to say that in his naïveté, which was genuine, he made a serious error, as often happened when he judged men and contingencies. In fact, the political professors he mentioned remained in the university, and I believed it was my duty to leave it. Thus, of about 400 signers of the Croce Manifesto, only 11 honoured their signature'.

On 3 November 1931 all of the professors of the University of Rome – including Volterra – received a memorandum from the rector Pietro De Francisci, who requested them to swear allegiance according to the following formula: 'I swear to be faithful to the King, to his Royal successors and to the Fascist Regime, to observe faithfully the Statute and the other laws of the State, to exercise the office of teacher and fulfil all my academic duties with the aim of forming citizens who are industrious, upright and devoted to the Nation and to the Fascist Regime. I swear that I do not belong to nor will I belong to associations or parties whose activities are irreconcilable with the duties of my office'. To the next memorandum, which informed the professors of the date of swearing, Volterra replied the same day: 'most illustrious rector my political ideas are well known inasmuch as they are made evident exclusively by my conduct in the Parliamentary arena, which cannot in any case be censured under article 51 of the fundamental Statute of the Kingdom. You will, esteemed sir, then understand why I cannot in good conscience accept the invitation sent to me in letter 18 of this month regarding the swearing of professors'. The tone is terse. There is no particular protest and no declaration, except perhaps that of trampled dignity. Volterra knew that he was facing immediate social downfall and ostracism. But just the same, he couldn't bring himself to utter the oath. Luigi Errera commented, 'you cannot believe how satisfied I am with this act, which apart from all else only confirms the esteem I have always felt for you character. The University, to its disgrace, will probably lose an illustrious teacher, but fortunately the *Maestro* remains, world famous, and is an example that we hope will not be lost for the future'.

Tullio Levi-Città[9] (1873–1941) tried in vain to obtain from the Faculty of Science a formal farewell and thank you for the man who had been at its helm for no fewer than 12 years and who would have, in different times, been named a

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[9] On the stance taken by Levi-Città, see Angelo Guerraggio and Pietro Nastasi, *Matematica in camicia nera*, op. cit. For an overall portrait of Levi-Città, see the monograph issue of *Lettera Matematica PRISTEM* (no. 57–58, 2006) edited by Pietro Nastasi and Rossana Tazzioli.

professor emeritus: 'I wish to express, at least for my own part and that of some mathematical friends, my sincere regret for the expulsion, just communicated to us, of Sen. Volterra from our Faculty, which was honoured with his scientific work for more than 30 years. The Faculty repeatedly showed its appreciation of Volterra's merit; not only, it also had the indirect moral advantage of the recognitions, truly exceptional in both importance and number, that Volterra received from the Universities, Academies and scientific institutions throughout the world'. On 31 December 1931, Volterra was dismissed from service and invited to request a pension, since he had the right to it. Many professors who were hostile to Fascism were convinced to swear allegiance by Croce and Einaudi, who recommended a Nicodemian attitude of prudence in order to avoid the danger of university chairs falling into the hands of those 'most ready to poison students' minds'. Volterra himself, apart from all else, took care not to encourage his eldest son Edoardo to refuse to take the oath. Edoardo, who had been beaten by a gang of fascists when he was a university student, was by that time a law professor in Rome. He later joined the Resistance, and went on to become a constitutional judge in the Italian Republic.

In the end, only 12 professors[10] in all of Italy refused to swear the oath of allegiance to the Fascist Regime: 'in them could not be discerned in any perceptible measure a design of common mobilisation, nor a hope of speaking to the vast masses or reaching agreement with the ranks of colleagues who agreed to swear. Each of them is immune to the hope that his action may give birth, in the present, to results that are perceptible or even vaguely anticipatory of victorious gains. . . . And nevertheless – for those who have followed their lives for a time – there arises the certainty that a loud and clear "no" like the one they have pronounced cannot be uttered without a secret joy'.[11]

In the meantime, one last oath was looming on the horizon for Volterra, who was by then more than 70. With the reform of academies and institutions for learning, enacted in 1933 to complete the process of fascistisation, the obligation to swear allegiance to Fascism was extended to the members of all the academies in the Kingdom. In June 1934 Volterra was once again invited to take an oath. Again he refused, and was declared expelled from all Italian academies, including the Accademia dei Lincei. Yet another time, he was one of a small but honourable number: among his Roman colleagues, in addition to De Sanctis as before, were the chemist Emanuele Paternò (who had already retired before the oath of 1931) and Benedetto Croce.

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[10] The 12 who refused to take the oath were: Ernesto Buonaiuti, Mario Carrara, Gaetano De Sanctis, Giorgio Errera, Giorgio Levi Della Vida, Fabio Luzzatto, Pietro Martinetti, Bartolo Nigrisoli, Edoardo Ruffini, Francesco Ruffini, Lionello Venturi and Vito Volterra.

[11] G. Boatti, *Preferirei di no*, op. cit.

Chapter 8
After the Cats, Fish

8.1 The Great Exception

In Chap. 5 and the war years, we spoke very little of mathematics. In effect, Volterra's political and academic responsibilities amply justified the fact that he had less time for research, and so there was less mathematical work for us to discuss. The same can be said for the period between the two world wars, with the one 'great exception' that we will deal with in this chapter.

It is said that mathematics is a young science, in the sense that mathematicians usually give the best of themselves while they are still young. It is no coincidence that the Fields Medal, which is the equivalent of a Nobel Prize for mathematics, is awarded to the most creative contributions among the contenders, and starting in 1966 the maximum age for winners was fixed at 40. In the period between the two wars, the Bourbaki group, which would have such a great part in the development of twentieth-century mathematics, adopted an internal rule so that when their members reached the age of 50, they were warmly invited to leave the group, the idea being that they had exhausted their most creative vein. The *topos* of a young mathematics is naturally opposed by those who have passed the threshold of 40 or even 50 years but nevertheless remain hopeful of still making original contributions. Instead, analogous but opposite reasons are cited by those who yearn to shrug off the encumbrances of older generations.

Without entering into the merits of either side, we cannot help but remark that by the period we are discussing here Volterra was over 60. To the inevitable scientific weariness – and this much at least we can concede to him – was added that of a political and civil life that was more intense than ever. We have already recalled his efforts in previous chapters: the foundation of the Italian Society for the Progress of the Sciences, the nomination as Senator, the presidency of numerous scientific societies, the war, the reorganisation of Italian research, the creation of the Italian National Research Council, the presidency of that organisation as well as that of the Accademia dei Lincei, the international congresses, the journeys abroad, the lectures in prestigious universities, and so forth. By that time, during the 1920s

and 1930s, it was natural that his more properly scientific work would feel the effects of this rhythm, in terms of quantity as well as in creativity, not least because some of these activities were still ongoing. It is therefore not surprising if, scanning the list of publications, we find above all texts of lectures and lessons, the reworking of arguments dealt with earlier, historical summaries, public and official speeches, and commemorative addresses.

One of the papers of a historical nature is the text of the lectures on 'Le calcul des variations, son évolution et ses progrès, son rôle dans le physique mathématique' (The calculus of variations, its evolution and progress, its role in mathematical physics) given in Prague and Brno in 1931. This is not so much a detailed history of the calculus of variations as a reconstruction of its main conceptual stages. Volterra begins by considering the work of Jacques Bernoulli, Euler, Lagrange, Legendre, Jacobi and Weierstrass, but dwells above all on more recent contributions. Now, the calculus of variations can be considered a chapter of functional analysis, limited to the consideration of those functionals that are expressed in the form of definite integrals and in search of their maximizing or minimizing curves. The instrument that made possible the new collocation was provided by the *direct methods*, strongly based on the notion of semi-continuity. This gives Volterra a chance to recall the contribution of the French mathematician René Baire, who had studied with him in Torino. The direct methods break the analogy with the classic methods of optimisation of real analysis, but in this way it is possible to avoid recourse to Euler's equation, its solution and the search for sufficient conditions.

In addition to the 'great exception', which we will deal with in a moment, there were also three monographs published by Volterra during the years between the wars. The first two are the books entitled *Leçons sur le composition et les fonctions permutables*, published in 1924 and *Théorie générale des fontionnelles*, published in 1936, both written in collaboration with Joseph Pérès (1890–1962).

Pérès, who taught in the universities of Toulouse, Strasbourg, Marseille and at the Sorbonne, had earned his degree at the Sorbonne in 1911. He had come to know Volterra during one of his visits to Paris, and had thus nurtured the idea of going to study in Rome: 'I still have the plan, about which I spoke to you in Paris, to spend some time next year studying with you, at the University of Rome .... Mr. Borel told me that my request is more likely to be granted if you support it'. This is from the first letter Pérès wrote to Volterra, dated 15 May 1912. The young French mathematician would actually complete his doctorate thesis in Rome.

Volterra had not had time to build a 'school', but he freely offered advise and suggestions to the young colleagues who intended to study under his guidance. Often these relationships gradually turned into collaborations and friendships. We have already had reason to mention Baire and Evans, and this is undoubtedly true also in the case of Pérès, as shown by his scientific interests – modelled on those of Volterra – and their correspondence was carried on over a remarkably long period of time. The last letter of Pérès to Volterra was dated 11 October 1940, just a few days before Volterra's death.

Between 1915 and 1916, that correspondence also included mention of the events of the war. In the letter dated 15 April 1915, Pérès stressed (writing in Italian) 'the

close ties between Italy and France, ties that perhaps before long will be shown on the battlefield'. Later, on 11 June, he expressed 'his complete joy' over the Italian intervention – which in his opinion was well planned both militarily and diplomatically – to uphold 'the union of all our forces against the barbarians'. Volterra's feelings are recorded in two letters, also dated 1915, in which he reiterated his interpretation of the war as a conflict between 'civility' and 'barbarousness', and thus that he could not but see united on the same side those 'who love civilisation and humanity'. Then, with the passing of years, the contents of their letters increasingly regarded mathematical projects and their development. Aside from a very discreet mention in a letter by Pérès written late in 1931 of Volterra's problems with the Fascists, which he had read about in the newspapers, expressing his total support and that of the French mathematicians, the letters constitute an extremely long series of observations on the technical aspects of the various projects for publication.

The first reference to the monograph of 1924 is by Pérès, in a letter of 18 February 1920: 'Mr Borel has in effect already said that you had thought of me for a work in common'. Pérès's contribution to this volume is rather modest. The book takes up topics which had already been amply dealt with by Volterra, enriched by several observations and formal simplifications introduced by Pérès. Instead, he took a more active role in the organisation of the volume on functionals. They began to discuss this in the summer of 1930: Pérès enthusiastically accepted Volterra's offer to collaboration in what was seen to be the 'great' conclusive work of several decades of research on functionals; it was Pérès's idea to divide it into two or three volumes.

What appeared in 1936 was only the first of the three volumes planned. It is divided into two parts: the first is dedicated to the theory of functionals; the second to integral equations. The chapter on the calculus of variations was written in large part by Leonida Tonelli (1885–1946), the greatest Italian expert in that field, who was also much appreciated for 'his' direct methods and his two-volume work on *Fondamenti del calcolo delle variazioni* (Foundations of the calculus of variations) published in Bologna in the early 1920s. In the summer of 1933, it appeared that Tonelli's health would prevent him for drafting the chapter assigned to him, and thus his place was taken by a student, Silvio Cinquini. Later, the situation improved and Volterra and Tonelli[1] were able to go back to the original plan. But aside from this chapter, the topics addressed in the first volume do not include anything particularly new. If anything, it provides a convinced presentation of some methodological foundations. There is an emphasis on the principal of the *passage from discrete to continuous*, now formulated with a series of 'rules' that can be followed to good end: to pass from a problem of ordinary analysis to a problem of functional calculus 'it will in fact be sufficient to replace a discontinuous index i *by a continuous index, or parameter, t*, and the sum with respect to this index i by the integral with respect to

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[1] The relationship between Volterra and Tonelli was very cordial. At the beginning of the 1920s Volterra campaigned (unsuccessfully) to have Tonelli 'called' to the University of Rome. Tonelli left the University of Bologna in 1930, accepting Gentile's invitation to move to Pisa.

the variable of integration t.[2] Both the choice of the topics dealt with and the expositive style itself provide confirmation of a 'concrete' functional analysis whose development is guided by the problems to which it is applied, rather than by an unconditional taste for generality. In short, nothing new. These are the same things that had already come out the notes written during the periods in Pisa and Torino. However, in the meantime there had also been the thesis by Fréchet (which we have already mentioned) and above all that of 1920 by the Polish mathematician Stefan Banach (1892–1945) with the specification of the concept of normed space and an ulterior, decided, acceleration towards what would become the 'modern' functional analysis of the twentieth century. The discipline became autonomous, and studied abstract structures; it no longer needed 'concrete' reasons to legitimise it. The very style of exposition which gradually imposed itself on writings in functional analysis grew further and further away from what Volterra had accustomed us to. Some fear of an exasperated formalism had to have in any case insinuated itself if Fréchet had to intervene on occasion against the tendency 'to pose arbitrary problems for the pleasure of solving them' and reducing mathematical research to 'a meaningless mind game'. In addition to Banach's thesis, in the 1920s and the early 1930s there appeared other important books, such as those of Paul Lévy, Marshall Harvey Stone and John von Neumann. Banach himself had published his book *Théorie des opérations lineaires*. Volterra showed himself to be sufficiently up-to-date on the most recent literature. Both the text and the bibliography of his 1936 publication include new references to the study of some general structures, but in the end the functional spaces dealt with remain the same 'concrete' and specific ones of his early works in functional analysis. The 'shape' of the work is the same. Volterra was elastic enough to recognise and accept the existence of a more general analysis, but he continued to think of functionals as a particular case that stands out in this generality. His functions of lines conserve their specific nature, which keeps them from being considered as mere examples.

Only the drafts of the second volume planned with Pérès remain. These deal with the theory of the composition of two functions, integro-differential equations, equations with functional derivatives, and the application of functionals to a generalisation of the theory of analytic functions. The third volume never moved past the preliminary discussion phase. On more than one occasion Pérès suggested that Volterra also cite the works of the Polish school of analysis. For his part, Volterra thought that it better advised to give more space to the results of Tonelli and Luigi Fantappiè,[3] avoiding in any case the risk of fragmenting the discussion into an excessive number of examples. A letter of 24 February 1940 states that, in this viewpoint, even those references to economics that were mentioned in the foreword to the first volume and which Volterra himself had initially insisted on,

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[2] *Theory of Functionals and of Integral and Integro-Differential Equations* (Mineola, NY: Dover Publications, 1959), p. 3.

[3] Luigi Fantappiè (1901–1956), a student of Volterra and Severi, taught at the universities of Cagliari, Palermo, Bologna and Rome.

ought to be sacrificed. This is a passage from the letter dated 10 February 1936: 'I'm starting to think about the third volume. I believe it will be better if confined to a small number of interesting applications rather a large number of applications in detail. I think it could be limited to applications: (1) the calculus of variations, (2) hereditary phenomena, (3) hereditary energy, (4) analytical functionals'.

Volterra and Pérès were not able to carry out their project. For Volterra, time was running out. He was, however, able to see published in 1938 his monograph, *Opérations infinitésimales lineaires. Applications aux équations différentielles et fonctionnelles*, written with Bohuslav Hostinsky (1884–1951). This book as well encompasses Volterra's *anciennes études*, integrated with more recent findings by Hostinsky regarding applications to physics and probability calculus. Volterra had met the young Czech mathematician in Paris in 1909. He met up with him again at the international congress in Strasbourg in 1920. It was Hostinsky who invited Volterra to Prague and Brno to give the lectures on the history of the calculus of variations that we mentioned earlier.

8.2   Volterra the Biomathematician

And at last we come to the 'great exception'. At the very moment that the inexorable passing of the years might lead one to think that Volterra's most creative phase was behind him, he succeeded in working out a new theory which all by itself would have been sufficient to assure him of a solid place in the history of twentieth-century mathematics. The great study of the 1920s and 1930s regarded the mechanisms of interaction between biological species cohabiting within the same ecosystem. This is how Volterra recalled its origins: 'as a result of conversations with Monsieur D'Ancona, who asked if we could find some mathematical way to study variations in the composition of biological associations, I started my research on this subject at the end of 1925'. He described this in a bit more detail elsewhere: 'Dr. Umberto D'Ancona entertained me several times with statistics that he was compiling about fishing during the period of the war and in periods previous to that, asking me if it would be possible to provide an mathematical explanation of the results that were obtained regarding the percentage of the various species in these different periods. ... This may justify my having permitted myself to publish this research, which is simple from an analytical point of view but which was new to me'.

This 'Monsieur D'Ancona' is the biologist Umberto D'Ancona (1896–1964) who had studied at the University of Budapest and later in that of Rome, where he earned a degree in natural sciences in 1920. He had also begun to publish, in the proceedings of the Thalassographic Committee, reports of some of his research work on the ichthyic populations of the Italian seas, that is, the characteristics of the populations of different species of fish. In 1930 he transferred to the University of Siena, then to Pisa, and finally to Padua, where he would remain until his death. What is most important, and most pertinent for our story, is that in 1926 he married Luisa, the eldest daughter of Virginia and Vito Volterra. Although not even

30 years had passed since their marriage, Italian society and customs had changed quickly. Luisa had finished her university studies and was a biologist: in intellectual bourgeois families, even the daughters were beginning to earn degrees and decide for themselves who to marry and what kind of future they would have!

But let's get back to Luisa's father. The quotations above were taken respectively from the *Leçons sur la théorie mathématique de la lutte pour la vie*, given at the Institut Poincaré during the 1928–1929 academic year and published in Paris in 1931, and from the article 'Variazioni e fluttuazioni del numero d'individui in specie animali conviventi' (Variations and fluctuations in the number of individuals of cohabiting animal species) published in 1927.[4] The French publication, which at its very beginning recalls the contribution made by Poincarè with the qualitative study of differential equations, marks the first phase of Volterra's biomathematical studies; the *Leçons* is a systematic and in-depth collection of the research that he had begun to publish in the form of several notes presented to the Accademia dei Lincei and the article of 1927 published in the proceedings of the Thalassographic committee. These studies had a decisive influence on the opening of one of the most productive lines of research in modern mathematical ecology, and would nourish the work of entire generations of biomathematicians. Thanks in particular to the *Leçons*, Volterra gained international acclaim in this field as well, leading him to become one of the best known figures in what has come to be considered the 'golden age' of theoretical ecology.

Volterra the biomathematician: after all that we have said about his personality and his 'natural scientific curiosity', the opening of a new front of investigation is not as surprising as it might seem. One of the reasons may have been his conversations with his son-in-law and the problem that he presented to him. But there is an entire earlier thread, which weaves together the inaugural lecture of 1901, the creation of the Thalassographic Committee, his interest in the Institutes of Marine Biology in Naples and Messina, and many of the battles in Parliament fought in the name of ecology.

To be specific, at the centre of the conversations with D'Ancona was the practical problem brought to light by some of the statistics compiled regarding the ports of the northern Adriatic Sea and its fishing industry. The interest in this data derived from the growing debate over the limited nature of marine resources and the relationships between fishing and fish populations. To be sure, this debate was not a new one, but the novel aspect of it was, for example, that the widespread use of motorised fishing boats made it possible to harvest fish in a way that was undoubtedly more 'efficient'. Is it true that an excess of fishing compromises the environment and the equilibrium of marine populations? Is it possible to prove statistically that an interruption, or at least a decrease in the harvest, even short term, leads to a significant increase in fish populations? The data that D'Ancona had relative to the period 1905–1923 – which included the war years, when fishing was

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[4] Vito Volterra, "Variazioni e fluttuazioni del numero d'individui in specie animali conviventi", *Reale Commitato Talassografico Italiano*, Memoria cxxxi, Venice, 1927.

less intense – indicated that the 'productivity' of the individual fisherman remained substantially constant, while within the harvest itself there was a considerable increase (during the war years and those immediately after) in the percentage of fish 'belonging to the class of the *sélaciens*', or elasmobranchii, a class of fish that includes sharks, rays and skates. On the basis of this data, the decrease in fishing could have led to an increased population of predators, which have a lower economic value. These are conclusions which, if confirmed, would have obvious practical implications. The decision to limit and reduce fishing activities would turn out to be a bad one, for a variety of reasons: it would lead to a decreased harvest in the short term, no significant increase in the periods that followed, and in any case an abundance of fish with low commercial value.

What D'Ancona asked his father-in-law to do was to explain, from a mathematical point of view, a dynamic that was not yet completely understood, given that its sole dependence on external causes – in this case the decreased activity of fishermen, due to the outbreak of the war – was not completely convincing. Why did the increase regard only predators, the 'class of *sélaciens* which, particularly voracious, feed on other fish' and not those preyed on? How was it possible to explain the statistics, according to which 'a decrease in the intensity of destruction favours the most voracious species'?

The problem placed before Volterra fell into a more general context of the study of variations found in biological associations constituted of several species living in the same environment. In short, the problem does not concern only fish: 'in agriculture, we can see how useful it is to study fluctuations of certain plant parasites, when these are opposed by their own parasites'.[5] Further on in the course of one of the lectures, Volterra said: 'We have given a very brief overview of the mathematical calculations related to the struggle for existence and population fluctuations that depend on it. But we could not touch on the relationship existing between these studies and other scientific research works. There [is], for example, a branch of applied zoology that deals with the destruction of pests in agriculture. . . . [We] must include research on microbial control in which the metabolic products and their actions are of great importance . . . . These questions even lead us to consider branches of medicine. . . . The sociological sciences, finally, must not overlook the research that formed the subject of our conference'.[6]

The struggle for survival leads to the registration of numerical fluctuations that do not seem capable of being interpreted solely in relation to periodic external changes, such as the cycles of daylight and darkness, or those of the seasons: 'there exist certain periodic environmental circumstances such as, for example, those that depend on the changing of the seasons, which produce oscillations that are forced and of an external nature, in the number of individuals in the different species.

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[5] Vito Volterra, *Leçons sur la théorie mathématique de la lutte pour la vie* (Paris: Gauthier-Villars, 1931).

[6] Vito Volterra, "Applications des Mathématiques à la Biologie", *L'Enseignement mathématique*, 36 (1937), pp. 297–330.

These external periodic actions were the ones that were particularly studied from the point of view of statistics; but there are others of an internal nature that have their own periods: would these exist even if the external periodic changes were to cease, and which [the internal actions] overlap?'.[7] In short, what does the increase of predatory fish depend on?

8.3 The Solution to the Problem

In order to respond to the question posed by D'Ancona, Volterra put into practice what he had already theorised in his inaugural lecture of 1901 regarding the construction of *mathematical models*. The problem suggested to him the idea of dividing the entire marine population into two large classes: *prey* and *predators*. Further, more refined definitions of marine fauna were excluded for the moment. The number of prey and predators would vary over time: it was precisely the study of these variations, and their possible dependence on greater or lesser intensity of an external force, such as fishing, that lay at the centre of Volterra's attention (and that of his son-in-law). Let's use $x = x(t)$ to represent the number of prey, and $y = y(t)$ the number of predators. In order to use the instruments acquired by analysis and infinitesimal calculus, let's suppose that t is a continuous variable, and that the functions $x(t)$ and $y(t)$ are derivable. In the 1901 inaugural lecture, Volterra had underlined how, especially in the field of biometrics, mathematical investigations tended to show a preference for tools of statistics and probability. Instead, the construction of his model followed along channels that were rigorously deterministic.

In order to understand how population x evolved over time, Volterra initially presumed that it was isolated. If it lived alone and coexisted with the other species without there being any mutual influence, direct or indirect, the number of births and deaths at each instant of an interval of time dt would be proportional to the number of individuals existing at that instant. We would thus have $dx = ax\,dt$, that is, $x' = ax$, and the evolution of the population would follow the well-known exponential law. The presence of predators modifies this dynamic, since the evolution of population x now depends on the number of encounters, within the interval of time, between prey and predators. Supposing in a first approximation that such a dependence is linear, we come up with the equation $x' = ax - bxy$ with a and b positive coefficients.

Volterra made analogous considerations with regard to the rate of growth of the population of predators and the evolution of its number $y = y(t)$. Again on the basis of the same hypotheses, we have the equation $y' = -cy + dxy$ with c and d positive coefficients (the change in sign is due to the fact that we are now speaking of the predators: in the absence of prey, and thus of food, these are destined to die out,

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[7] Vito Volterra, "Una teoria matematica sulla lotta per l'esistenza", *Scientia*, 41 (1927), pp. 85–102.

while, by their very nature, encounters with prey are a positive factor). The quadratic model is therefore composed of the following system of first order differential equations:

$$\begin{cases} x' = ax - bxy \\ y' = -cy + dxy \end{cases}$$

with the given initial conditions $x(0) = x_0$ and $y(0) = y_0$. The constants a and $-c$ represent the growth coefficients of the two species, in the absence of any interaction whatsoever; the coefficients $-b$ and d measure the influence of each species on the growth rate of the other.

The system is not particularly difficult to solve. By dividing up the equations of the system expression by expression, it is possible to obtain an equation in which the variables are separated:

$$\frac{dy}{dx} = \frac{y(dx - c)}{x(a - by)}$$

and thus (in implicit form) the solution is $y^a e^{-by} = kx^{-c}e^{dx}$, with k dependent on the initial conditions and on the parameters of the system.

The technique used to draw the trajectory C in the plane of the phases, while simple, is particularly ingenious. If we set $z = y^a e^{-by}$ and $w = kx^{-c}e^{dx}$, the problem reverts back to a search for the points (x, y) of the plane such that $z = w$. At this point, Volterra used a system of reference with four axes and found that C is a closed curve. As the parameters a, b, c, d, x_0 and y_0 vary, the orbit expands or contracts, defining a family of ovaloids about a point $S = (c/d, a/b)$ whose coordinates play a particular role. The values of $x = c/d$ and $y = a/b$ are relative to populations in equilibrium: if at a certain instant the populations reach these levels, then successive values of $x = x(t)$ and $y = y(t)$ remain constant.

The particular geometric configuration of curve C expresses the cyclic (or periodic) evolution of the two populations x and y. In other words, the endogenous factors are sufficient – seeing as how Volterra had programmatically excluded up to this point any environmental interaction whatsoever – to justify the fluctuations in the number of prey and predators and their periodic oscillations. For its part, the mathematical solution also had a biological interpretation as well: at the beginning, when we can suppose that the levels are low for both populations, the number of predators is reduced so low as to favour the development of the prey; the predators continue to diminish, given the scarcity of encounters, although the rate of decrease becomes progressively slower until it reaches a minimum, which marks the inversion of the trend; from this moment, they begin to increase in virtue of a sufficiently high number of encounters, while the growth rate of the prey diminishes gradually, until it reaches a maximum, which marks a new inversion of the trend. At this point the cycle begins all over again, and so forth.
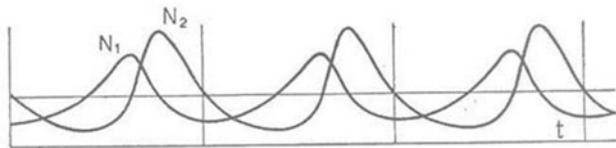
What remains to be understood is the "fishing effect" and the reasons why a decrease in fishing activity during the war years and immediately after led to an increase in the number of predators. To this end, knowing at this point that the cyclicity of the population dynamics defines a period T such that $x(t+T) = x(t)$ and $y(t+T) = y(t)$, Volterra calculated the average number of individuals present over the arc of a period:

$$\bar{x} = \frac{1}{T} \int_0^T x(t)dt, \quad \bar{y} = \frac{1}{T} \int_0^T y(t)dt$$

Dividing the equations of the initial system by x and y respectively, he obtained:

$$\frac{x'}{x} = \frac{d \log x}{dt} = a - by$$

from which, integrating between 0 and T and recalling that $x(0) = x(T)$, $y(0) = y(T)$:

$$0 = aT - b \int_0^T y(t)dt$$

$$0 = -cT + d \int_0^T x(t)dt.$$

In conclusion, the coordinates of the centre of the ovaloid express the average number of individuals of the two populations present over the arc of a period, and this independent of the particular initial conditions: 'averages over a period of the numbers of individuals of both species are independent of initial conditions, and equal to the numbers that correspond to the state of equilibrium'.

Volterra's model now made possible a quick explanation of the 'anomaly' that his son-in-law had presented to him. In general, the consequence of fishing will be a decrease in the two populations in the measure of $sx(t)$ and $sy(t)$, where the constant s reflects the intensity of fishing. The new situation is described by the following (modified) system of differential equations:

$$\begin{cases} x' = ax - bxy - sx = (a - s)x - bsy \\ y' = -cy + dxy - sy = -(c + s)y + dxy. \end{cases}$$

This system is identical in structure to the previous one, with a and c substituted by $(a - s)$ and $(c + s)$ respectively. We can then immediately confirm that the average values are given by:

$$\bar{x} = \frac{c + s}{d}, \;\; \bar{y} = \frac{a - s}{b},$$

which provides confirmation of the empirical observation that a certain decrease in fishing activity leads to an increase in the population of predators and a decrease in that of prey: 'if you destroy the two species uniformly and in proportion to their numbers of individuals (at a low enough level so that the fluctuations are maintained), the average number of individuals of the consumed species increases while that of the consuming species decreases'!

8.4   Other Developments

The model analysed up to this point is the one that responded to the question about the statistics regarding fishing in the ports of Trieste, Fiume and Venice, and would pave the way for Volterra's studies on population dynamics. But even before that, later chapters of his *Leçons* included several extensions of this.

The first generalisation regarded the phenomenon of predation in complex ecosystems that were characterised by not just two but any number of species. The system is now one of "generalised predation" with n species that nourish or are nourished by each other. There is obviously a degree of further formal complication, but above all the mathematical treatment – in particular, the consideration of a matrix and the calculation of its determinant – lead to distinguishing the cases in which n is even or odd. This is undoubtedly a critical moment for the model and its capacity for interpreting the phenomena, since the description of the ecosystem comes to depend on a condition that is strictly mathematical. In the two cases, the situation is presented in terms that are effectively different. For the case where n is even, Volterra succeeded in extending the laws worked out for two species, but for the case where n is odd, it is no longer true that all the populations remain limited between two positive numbers: the system loses its equilibrium and the population of at least one species will tend to either die out or to explode.

Then, in Chap. 3 of the *Leçons* Volterra introduces a distinction between conservative associations (when it is supposed that 'the encounters between individuals of the various species do not alter the value of the biological association') and the more realistic, dissipative associations: 'the case of conservative associations might be considered a limit case which is approached by associations in nature, but even closer to the systems that effectively exist appear to be the dissipative associations, the value of which is diminished by each encounter between individuals of two species that results in a consumption'. The distinction is introduced in the study of the sign of a quadratic form, but its physical matrix is

evident, starting with the terminology used. Underlying all of Volterra's contributions to biomathematics is that tendency towards reductionism that we saw at work when we spoke of mathematical economics and commented on the inaugural lecture of 1901. His faith in the validity and conceptual apparatus of mechanics is such that Volterra intended to use the fundamental principles in the study of biological phenomena as well. The development of every discipline is related back – that is, *reduced* – to what was learned from the history of the first theory to be mathematised. Mechanics represented the guiding light for all other disciplines, which were modelled to resemble it almost as though this would make it possible to build, in the field of biomathematics, a rational mechanics for populations. At a certain point, in the 1937 article published in *L'Enseignement mathématique*, Volterra wrote: 'all the conclusions drawn in mechanics from Hamilton's principle can be transposed into the field of biology'. It is therefore no surprise that the third chapter of the *Leçons* is followed by a final section dedicated to extending the previous models to the consideration of the dynamics of heredity. The title of the chapter is already significant: 'Sur les actions héréditaires comparées en biologie et en mécanique' (On the hereditary effects that appear in biology and in mechanics). This is an opportunity to use integro-differential equations. It is the opportunity for us to see how mechanics remained a constant point of reference for Volterra, in a working life that has in any case been characterised by continuous adaptations. Mechanistic determinism is also one of the systems with a memory, particularly suitable for studying the phenomena of life that show peculiarities which must be taken into account in order 'to know the noteworthy and sometimes preponderant influence of the past to determine the reactions in the present and the evolution of the future'.

We mentioned the great fame that Volterra acquired, now in the field of biomathematics as well, thanks to the publication of his *Leçons sur la théorie mathématique de la lutte pour la vie*. From the letters exchanged with Umberto D'Ancona we learn that the expositive form adopted had been the object of serious reflection in order to make the presentation accessible to the widest readership possible. D'Ancona wrote to Volterra on 21 June 1926, 'With regard to the re-publication of the paper for the Thalassographic [Committee], as I had already proposed to you, it would be a good idea to put it in plainer form for the use of biologists. The paper for the Lincei is alright as it is because it will be read by mathematicians as well, while instead that of the Thalassographic [Committee] will mainly be read by biologists. And with regard to the French publication: 'if the book is to be destined for naturalists as well, then it is a good idea, I think, to distribute the material so that the naturalists can read without being encumbered by too much calculus. Perhaps the material could be arranged by putting alternating chapters or passages in a different font, or by adding notes so that those who wish to do so can skip the part of calculus used to find a given formula but can still take the formula itself. For readers who are not mathematicians it's a good idea instead to add some elementary explanation of the way to apply the formulas themselves, giving examples as well. Perhaps in this way it is possible to make a book that is useful for mathematicians and for naturalists. Otherwise I fear that the naturalists,

finding themselves faced with a long string of calculations, will lose the thread of reasoning and in the midst of numerous formulas won't know which to apply nor how to apply it'.[8] In spite of these suggestions and similar worries on the part of Volterra, the *Leçons* remains a mathematics book aimed at those who, all told, already possess a certain degree of mathematical knowledge. As often happens, the technical notes – added at the end of the different chapters by Marcel Brelot, 'an old student at the Scuola Normale Superior' turn out to be an apparatus whose usefulness is doubtful, to say the least. Even Volterra was of that opinion, as we can see from his letter to D'Ancona of 28 March 1930: 'I am sorry that you are not satisfied with the edition of the French book. I believed that it was you who had had them put those notes on mathematics for the use of mathematicians. To tell the truth I don't believe that a naturalist will find this French edition very much clearer than the earlier Italian papers. Those were comprehensible in their findings even for the naturalists. ... In order for the naturalists to be able to understand completely how the operations work it would be necessary to state beforehand the elements of calculus and that would mean making a treatise instead of a paper. Or it would be necessary to expound everything in a more elementary way, which doesn't seem to me to be possible. ... But what Mr Brelot did doesn't seem to me either to be the best way to make the paper accessible to the naturalists'.

But still, the echo was remarkable. The book's success was helped by the name of its author, by its compilation in a language that made it accessible to an international readership, and by the originality of the subject matter and the mathematical instruments used. Mathematics was not limited to a role that was purely computational, or to translating into formulas what had already been learned from experimental observation and common sense, but rather made it possible to arrive at conclusions that in some cases were truly original. Of further significance for understanding the book's positive reception, was the particular liveliness of the scientific and cultural spheres that were producing the most recent theories on ecology and evolution.[9]

Volterra himself grew passionately interested in the subject, and biomathematical topics were substantially the only ones that he would work on during the second half of the 1930s. But he had immediately begun an intense campaign to disseminate and popularise the subject, as shown by numerous lectures and talks at congresses he gave, as by two articles published in *Nature*[10] and *Scientia*.[11] He then began corresponding with a series of American researchers, and with the French Jean Régnier (1892–1946) and the Russian Georgii Gausse (1910–1986).

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[8] Letter from Umberto D'Ancona to Vito Volterra, 10 February 1929.

[9] See Piero Manfredi and Giuseppe A. Micheli, "Ecologia matematica e Matematica delle popolazioni", in *La matematica italiana dopo l'Unità. Gli anni tra le due guerre mondiali* (Milan: Marcos y Marcos: 1998), pp. 671–734.

[10] Vito Volterra, "Fluctuations in the abundance of a species considered mathematically", *Nature*, 118 (1926), pp. 558–560.

[11] "Une thérie mathématique de la lutte pour la vie", *Scientia*, 41 (1927), pp. 33–48.

However, his most important correspondent in the field of biomathematics was the biologist and mathematician Vladimir Kostitzin (1882–1963?),[12] first the director of the Moscow Geophysics Institute and later an exile in Paris. Volterra even collaborated with him on the script of a film, entitled 'Immagini matematiche della lotta per la vita' (Mathematical images of the struggle for existence), which was shown for the first time in Paris, at the Palais de la Découverte, in 1937.

Two years earlier, in 1935, Volterra had published, again with D'Ancona, the monograph entitled *Le associations biologiques étudiées au point du vue mathématique*, which contained several new things concerning the theoretical investigation and paid greater attention in general to the connection to the aspect of application. In 1938 and in 1939 he would publish two more papers on the logistic growth of a population.

The consensus concerning his theories and the methodologies followed is remarkable. A mathematician of the calibre of Andrej Kolmogorov (1903–1987) would intervene on the model that we have described above. The observation that 'the analytical expressions chosen by Volterra for the second expressions of the equations . . . can be considered only as the first approximations of the actual state of things'[13] led him to generalise Volterra's study on the 'reciprocal actions of an eaten species and an eating species' with the system:

$$\begin{cases} x' = K_1(x, y)x \\ y' = K_2(x, y)y. \end{cases}$$

and various hypotheses on the functions K_1 and K_2.

Volterra's work also gave rise to a significant quantity of critical debate. There was first of all the question of priority involving the American physicist and chemist Alfred Lotka, who, in his *Elements of Physical Biology* of 1925, published just a few months before Volterra's first papers on the subject, had constructed a model that was formally analogous. Volterra did not cite his work in his papers because he didn't know it. Later he had no difficulty acknowledging the merits of his colleague, although he immediately emphasised some of their differences: 'I am sorry that I wasn't able to cite the work by Dr. Lotka, which contains different applications of the mathematics to questions of chemistry and biology'.[14] The letters between Lotka and Volterra don't show any particular

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[12] For more about Kostitzin and his contributions, see Francesco M. Scudo, "Vito Volterra, Ecology and the Quantification of Darwinism", in *Atti del Convegno internazionale in memoreia di Vito Volterra,* op. cit.; see also Giorgio Israël and Ana Millán Gasca, *The Biology of Numbers* (Basel: Birkhäuser, 2002), which contains an interesting account of the debate that arose over Volterra's contributions.

[13] Andrej Kolmogoroff, "Sulla teoria di Volterra della lotta per l'esistenza", *Giornale dell'Istituto Italiano deli Attuari,* 7 (1936), pp. 74–80.

[14] Vito Volterra, "Variazioni e fluttuazioni del numero d'individui in specie animali conviventi", op. cit.

antagonism as much as they do a convergence towards the same model – the one that is now referred to as the Lotka-Volterra predator–prey model – created, however, for reasons, interests and methodologies that were quite different, even to the point of having very little in common.[15]

There were then the discussions with the biologists who sometimes reproached Volterra for having only scant knowledge of the specialised literature, and criticised some of his findings (in particular, the significance he gave to oscillatory behaviour), which, in their opinions, only corresponded slightly to reality. Volterra was also accused of having neglected a series of characteristics of individuals which are essential if the study is not to remain at a purely qualitative level, unconfirmed by numerical calculations and by comparison with experimental data. Volterra's counteroffensive[16] was based entirely on the concept of the mathematical model and how it is constructed, as described earlier in the 1901 inaugural lecture. He replied, 'in order to be able to treat the problem mathematically, it is opportune to begin with hypotheses which, although removed from reality, provide an approximate image of it. Even if the representation is, at least in the beginning, rough, as long as it is simple, whether is possible to apply calculus to it and check quantitatively or even qualitatively if the results obtained correspond to the statistical data and thus test the correctness of the initial hypotheses and prepare the terrain for new results'. He was even more explicit in the article that appeared in *Scientia*, going back to the reassuring analogy with the development of mechanics. The model's construction must take into account as far as possible the problem to be treated, its nature and particular characteristics, as well as the mathematical tools that will be employed.

The discussion of the results then makes it gradually possible to improve their meaningfulness: 'in the case treated, of two species, one of which eats the other, we find that there is always established a periodic cycle which makes the two species oscillate about a given mean value. It might be objected that it is easy to imagine the eating species to be so numerous and so voracious that in a brief time one by one all of the individuals of the other species are destroyed, thus rendering the aforementioned oscillations impossible. But we must point out that the law of the cycle descends from the proposition that a species lacking nourishment can only die out over an infinite time, and that this appears to be even further from reality than the law itself. That depends on the fact that among the hypotheses that lie at the base of the entire treatment is the one that the number of individuals is a positive number

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[15] On this question, see Giorgio Israël, "Le equazioni di Volterra e Lotka: una questione di priorità", in *Atti del Convegno "La storia delle matematiche in Italia"*, O. Montaldo and L. Grugnetti, ed. (Cagliari: Università di Cagliari, Istituti di Matematica della Facoltà di Scienze e Ingegneria, 1982), pp. 495–502.

[16] For a "modern" defence of the approach taken by Volterra, see, among others, Martin Braun, "Why the Percentage of Sharks Caught in the Mediterranean Sea Rose Dramatically During World War I", in *Differential Equation Models*, Martin Braun, Courtney S. Coleman, Donald A. Drew, eds. (Berlin: Springer-Verlag, 1983).

that varies continuously, while in reality this can only be a natural number and cannot go below one. . . . None of this is particular to the applications of mathematics to biology, but rather there are analogies present in all the other cases in which the continuous is substituted by the discontinuous. . . . How do we proceed, in theories that have by now for some time been applied, to overcome the difficulties we are dealing with here? It is opportune to distinguish two phases: in the first the problem is solved by abandoning ourselves, if you will, to the procedures of analysis, considering the hypotheses made as if they were absolutely true. Once the solution is obtained, then in a second phase, it is opportune to discuss them, and if in the solution it appears that certain facts are too far removed from reality, the solution must be either renounced or modified. . . . The solution found is not therefore in vain, but reveals to us a circumstance of remarkable importance and usefulness. The first phase we have described above constitutes what we might call the *rational phase*, the second, the *applied phase*, and we in fact have a *rational mechanics* and an *applied mechanics*. The research carried out, and briefly described here, in biological mathematics would appertain, according to this classification, to the *rational phase*'.

Chapter 9
Time Runs Out

9.1 In Shadow of St. Peter's

1933 witnessed not only the Fascistisation of the statutes of the university, but also Hitler's takeover of the German government and the onset of the persecution of the Jews. While in the beginning Mussolini's attitude was that of a spiritual father of the Nazi movement, it soon became clear that the hierarchy would not be determined by the number of years in power, but rather by the differences in economic and military might, which were sharply in favour of Germany. 1935 was then the year that Italy invaded Ethiopia, prompting the League of Nations to impose economic sanctions on it. The resulting international isolation led to a weakening of the pro-English sentiments of the Fascist regime, and a symmetrical strengthening of pro-German sympathies. The growing ties with Germany fed the racism and anti-Semitism that had been present in Fascism from the very beginning, but had not found any particular outlet up to then.

One discriminatory practice was however already evident in the nominations for the Accademia d'Italia. After the first roster of members nominated directly by the government in 1929, the new appointments were made in groups of three proposed by the academy's four classes: although a certain number of Jews were proposed in this way, none of them were ever selected by the Duce, who performed the final choice of those admitted. As a consequence, there was never a Jewish member of the academy, in spite of their significant numbers in the world of high culture in Italy.

In the mid-1930s, support for Fascism reached its height in Italian society. The colonial war and ensuing sanctions brought support for the Duce from even those who had been least certain. This political climate undoubtedly weighed heavily on Volterra, in spite of the apparent serenity described by his family members and correspondents. The man who had been 'Mr Italian Science', an esteemed and revered mathematician who had held the reins of power as president of the National Research Council and the Accademia dei Lincei (and whose power was not only academic), was by then more than 70 years old, afflicted by various aches and pains,

and some cardio-circulatory problems. He had been deprived of not only power, but also of many of the professional and personal relationships which, carried on into old age, are a warm part of the autumn of life. Melancholy over a past torn away from him so abruptly – in just 7 or 8 years – was somewhat mitigated by a few trips abroad; in 1937 he managed to visit Egypt, and in 1938 he went to Paris for what would turn out to be the last time. Virginia was always at his side. There were grandchildren. After Luisa's marriage to Umberto D'Ancona, Edoardo had also married, and had begun his university career in law. His wife, Nella Levi Mortera, left a brief but intense memoir of Volterra's life before and during the war.[1] 'Inebriated by success in Ethiopia, the Fascists lost even their sense of the ridiculous. Children were so longer children of their mothers, but of the Roman she-wolf, and the women had to be prolific; employees had to marry, the Fascist party officials had to jump through rings of fire, whatever their age and physical condition might be. While the newspapers enslaved to the government never tired of repeating that Italy was the master of water, air, East and West, in pubs and on trains ruffians and loudmouths were heard to sing at the top of their lungs lines like "The end of England begins in Jarabub" and "If England doesn't move it, we'll take Egypt too". The few thinking people gathered in some friend's house around the cakes made with vegetina[2] and barbarous karkadè and contented themselves with repeating the usual *bon mots*: What is the flower of Fascism? The magnolia. And that of anti-Fascism? The rhododendron. Who is the Duce? The flounder of the Empire. If you don't mysticate, you don't masticate.[3] And so on. One lady from Romagna commented on and concluded these gatherings with a proverb that was as expressive as it was filled with common sense: "Chatter doesn't make pancakes"[4] and she wasn't all wrong'.

   At a time as difficult both politically and morally as that, an unexpected expression of solidarity with Volterra came from a place he would never have dreamed of: the Vatican. On 28 October 1936, Pope Pius XI enacted the reform of the Pontificio Accademia dei Nuovi Lincei, reorganising it in a new form with a new name, Pontificia Accademia delle Scienze. This had all begun with the old Accademia dei Lincei (*lincei*, linxes, are famed for their sagacity) founded by Federico Cesi in 1603 and closed in 1630, at the death of its noble patron. Attempts to revive it came and went, until it was reconstituted by Pope Pius IX in 1847.

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[1] Nella Levi Mortera, *Ritorno alla libertà* (no bibliographical information given). This was a booklet that was published privately by her daughters Laura and Virginia.

[2] 'Vegetina' was a legume-based flour used in baking, invented in Germany and exported to Italy.

[3] These were well-known jokes. The one about 'magnolia' derived from the root 'magn-' that accompanied Mussolini, who was said to be the son of a 'magnano' (meaning ironmonger) and was 'magnanimo', or magnanimous, as well as playing on 'Magno' or great, as in 'Charlemagne'; the 'rododendro', rhododendrun, was the flower of the anti-Fascists because 'rodo dentro' means 'I am gnawing away inside'. The joke, 'if you don't mysticate, you don't masticate', that is, you don't eat, was a poke at Giovanni Gentile's book entitled *Mistica Fascista*, or Fascist Mysticism.

[4] This is a variation of the popular saying, 'Acqua e chiacchiere non fanno frittelle', water and chatter don't make pancakes, that is, idle talking doesn't lead to anything.

In 1870 the Accademia was split: part of the members accepted its transformation, promoted by Quintino Sella, into the Reale Accademia dei Lincei, of which we have heard so much, while others remained loyal to the Pope and maintained the former association, which had its seat in the Vatican gardens. After a period of difficult relations, the hostility between the two academies began to wane, ultimately becoming cordial, as can be seen from the correspondence between the last president of the Nuovi Lincei, Father Giuseppe Gianfranceschi, and Volterra. The new Pontificia Accademia delle Scienze continued the traditional activities carried on by all groups of this nature: discussions of scientific problems, presentations of communications, notes and papers, publications and organisation of congresses and meetings.

The first president of the new pontifical academy was Father Agostino Gemelli, a brilliant physician trained in the Pavian school of Camillo Golgi, who we mentioned briefly in Chap. 7 when he was president of the Society for the Progress of the Sciences, and whose conversion to Catholicism had caused a sensation. The by-laws called for 70 members, all nominated by the Pope. To prepare the roll of the members – the first roster presented to the Pope included 33 Italians and 37 foreigners – Gemelli called on an intimate group of collaborators, one of the main ones of which was Tullio Levi-Civita[5] (Figs. 9.1–9.3).

It was Levi-Civita who named Volterra, in a long letter dated 27 January 1936 regarding the new nominations: 'I attribute a special place to three people: Volterra, Enriques, Fermi. Senator Vito Volterra shares with Hilbert the first place among living mathematicians. If we look only at official recognitions, Volterra has to be considered superior, as he belongs to all the major scientific academies of the world, with no exceptions; he has been invited, on solemn occasions, to give lectures at universities and other scientific institutions in Europe and the Americas, etc. Without taking into consideration his by-now classic results, ... we need mention only his recent research in mathematical biology, which opened a new field of research for higher analysis. ... Volterra continues to produce in spite of his 75 years. As is known, having refused to swear the oath required by the Fascist government, he has been in retirement (from being professor at the University of Rome) now for 5 years, and was later ousted from the academies of which he was an active member (Lincei, XL, Torino, Naples)'. Of the possible new members suggested by Levi-Civita, only Vito Volterra was accepted by the Pope.[6] The Pope would also ask that special prayer be said for Volterra, when the aged mathematician was suffering health problems due partly to age, but also partly to the moral difficulties that Italy had to face. For his part, Monsignor Giovanni

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[5] The documentation about this is found in *Calendario della corrispondenza di Tullio Levi-Civita (1873–1941) con appendici di documenti inediti*, Pietro Nastasi and Rossana Tazzioli, eds. Quaderni PRISTEM, 8 (Milan: Università Bocconi, 1999).

[6] The other mathematicians nominated to membership in the pontifical academy were Ugo Amaldi, George David Birkhoff, Constantin Carathéodory, Charles-Jean de la Vallée Poussin, Tullio Levi-Civita, Émile Picard and Edmund Taylor Whittaker.

Fig. 9.1 Vito Volterra with
Tullio Levi-Civita
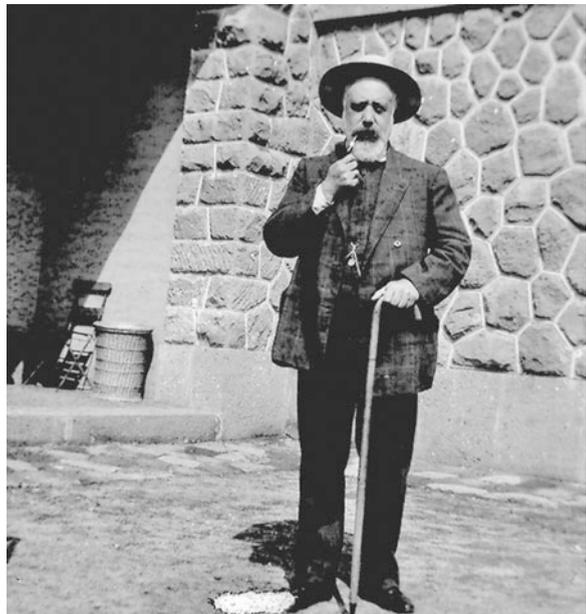
Fig. 9.2 Vito Volterra at his
home in Ariccia

Fig. 9.3 Vito Volterra with his wife Virginia and their grandchildren

Battista Montini, who was to become Pope Paul VI, took an interest in the solution of some questions that seriously worried the Volterra, in relation to the racial laws we are about to discuss. At the end of 1938, the Pontificia Accademia also gave Volterra the satisfaction of accepting his proposal for the nomination of Carlo Somigliana as a member; Somigliana had also been on Levi-Civita's list.

Some months later, Roberto Farinacci wrote to Mussolini to recommend that Father Gemelli be nominated a member of the Accademia d'Italia. The Duce's comment was peremptory: 'He is not yet mature'. The excess of attention towards the Jewish members of the Pontificia, and perhaps also his human solidarity with an anti-Fascism that was openly declared, certainly carried weight in this judgment. But it's an ill wind that blows no good: by 1940, recognition at that late date by the Fascist regime most likely would have done more harm than good to the ecclesiastic who counted the most in the Vatican's cultural organisation.

9.2   The Racial Laws

On 14 July 1938, the 'Manifesto of Race' declared that the people of Italy belonged to the Aryan race, and stripped the Jews of Italian citizenship. At the beginning of September, Royal Decree 1381 dictated the first measures to be taken with regard to the Jews; the decree that followed, 1390, outlined steps 'for the defence of the Italian race in Fascist schools'. Following deliberation by the Grand Council of Fascism on 6 October 1938, a long and detailed decree of 17 November 1938 introduced the 'Legislative measures for the defence of the Italian race'. The

various racist and anti-Semitic measures were finally coordinated into a single text on 15 December 1938.

The new legislation had various effects on the Jewish community in Italy, which at that time numbered about 50,000. In the space of just a few days, almost 4,000 people, between professors, public and private employees, professionals and merchants, were thrown out of work. The effects of the racial laws on the mathematical community were devastating. Excluded from teaching were mathematicians of the calibre of Guido Ascoli, Ettore Del Vecchio, Federigo Enriques, Gino Fano, Guido Fubini, Beppo Levi, Tullio Levi-Civita, Beniamino Segre and Alessandro Terracini. Those who could emigrated: almost 6,000 between 1938 and 1941. But leaving wasn't so easy: of Volterra's children, Enrico had to overcome many difficulties before being able to get out of Italy, while Edoardo and Nella (with their daughter, Laura) were forced, after many attempts, to give up and come back to Italy, where they lived in hiding.

Preceded by a hard-hitting campaign in the press, the racial laws were applied with particular fury in the world of culture. Already before their enactment, the racist newspaper *Il Tevere* directed by Telesio Interlandi had published a list of Jewish university professors and assistants, demanding their removal from universities and expulsion from academies. The lists used by Interlandi had been provided by the Ministry of Education on the basis of a census of Jews taken a few months before the enactment of the autumn 1938 laws, and were almost identical to the lists later used for their removal and expulsion. The racist press also anticipated the list of the textbooks by Jewish authors, whose use in schools was later prohibited.

Nella Levi Mortera recalls events this way, 'On 14 July 1938 the Manifesto of Race was published. Immediately afterwards began a violent anti-Semitic campaign. Listening to the *News of the Regime* on the radio, whose commentary *ad usum delphini* on the facts of the day, with the sole aim of swaying public opinion, opening a newspaper at that time and reading the truth altered and deformed, and the lies used in the attempt to sling mud at innocent people. . . . I can still see the crocheted crosses that began to appear in the shop windows of stores run by zealous Fascists. Then, little by little, the little pinpricks, being prohibited from entering the library, other more or less useless vexations. Get rid of the cleaning lady who might not be Aryan: she might contaminate us! Get rid of the radio! They might listen to Colonel Stevens![7] Then things became increasingly serious. Denaturalised Jews could not longer go to school, and a few months later the same fate awaited our children. Events came to a head, and the decrees of October were passed, with which those teachers who were not of the Aryan race were prohibited from teaching. If before you had been struck by people's incomprehension . . . now our greatest torment came from those who hurt our already

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[7] Colonel Stevens, known in Italy as 'Colonello Buonasera' was Colonel Harold Stevens of BBC London, who transmitted each evening in the Italian language, and opened his transmission by saying, 'Buonasera, qui Radio Londra'.

sensitive feelings, trying to console us with phrases like, "We can still be friends", or "I am sorry for you personally, but if Mussolini did it he must have had his reasons, no?", and "You're not Italian, what are you doing here?". The heinous propaganda began to bear fruit. Is it possible that so many brains can be clouded over to the point of no longer knowing how to reason, to understand that sacred principles have been violated and that this should be the cause of more worry and anguish than the personal fate of this or that friend of theirs?'.

In the attempt to sow spite and division within the Jewish community, the racial laws called for a mitigation of some of the measures with regard to those Jews who could claim the particular patriotic merit outlined by the laws. Thus, in the attempt to limit the damage to himself and his family, Volterra prepared a request for 'discrimination' – that was how the law defined the differentiation of those Jews to whom such benefits could be applied – but, fortunately, at the last moment he did not have to present it. He was able to avoid that humiliation thanks to a measure signed by the king, and communicated to Volterra in a letter from the president of the Senate, which officially extended 'discrimination' to all Jewish senators, whose patriotic merit was accepted as a matter of course since without it they would never have risen to the level of senator. Vittorio Emanuele III's legislation, first in support of the anti-Fascist senators and then towards the Jewish ones, as well as the discreet protection provided by the sovereign for several members of the military who were of the opposition, shouldn't mislead the reader regarding his substantial complicity with the Fascist government. The king only intended to impose some precise limits on the statutory distortions of Mussolini in order to safeguard his prerogatives, in the conviction that these might turn out to be useful should he decide to get rid of the burdensome 'head of the government and Duce of Fascism'. In fact, that political caution would show all of its usefulness on 25 July 1943, when Mussolini was forced to step down, and all its fragility the following 8 September, when US General Dwight Eisenhower publicly announced the surrender of Italy to the Allies. But going back to our story, on 19 December 1938, for some reason Volterra, who had not yet been stricken from the list of the Istituto Lombardo di Scienze e Lettere, received a communication informing him that he had ceased to be a part of that institute 'because he belonged to a non-Aryan race'.

In the spring of 1939, Volterra, by then 79, was old and worn out. He wrote to Carlo Somigliana, 'I am always seriously ill and am now suffering from phlebitis. I hope to see you next month when you come here, but I don't know if I will be able to take part in the session of the Accademia'. His friends have not forgotten him. Somigliana thanked him for the nomination to the Pontificia Accademia, and added, 'I also think, with sincere regret, of your sorrows and your worries for the future of your children, who began so brilliantly. What has come to pass in these most recent times has embittered all those who have a heart. And one wants to hope that this wind of folly will at last be tempered or stopped, at least before long. One must keep a strong soul and not be discouraged. This enormity will at last be followed by a return to sagacity'. Somigliana and Volterra had met at the Scuola Normale in Pisa 60 years earlier. Now they were in the time of memories. Volterra wrote to Somigliana: 'Having had to stop for a day in Florence, I took a trip to Montecatini,

to visit our old friend Bindi, who I hadn't seen for 58 years! To tell the truth, I found him a little changed from the handsome fellow he had been when we were in Pisa, but in very good shape. We recalled our old colleagues, and you first of all; of many we have had no more word'. The French mathematician Paul Montel wrote in December 1939, 'We will celebrate the jubilee of Borel on 14 January. The mighty voice of Volterra will not make itself heard, but we all know that his thoughts will be with us, Royal Majesty, absent but present, as D'Annunzio said in his speech at Quarto'. His son Edoardo, with his family, arrived in France on the day war was declared, but the arrangements made ran into such difficulties that they were forced to return to Italy: 'I decided . . . to bring the little girl back to Italy, where the war had not yet begun. Once back in France, after having entrusted my daughter to her grandparents, I immediately repented not keeping her with me, since Italy was right on the brink of entering the war, and we risked what I had always feared, that is, being separated from her for the duration of the conflict'. Italy entered the war on 10 June 1940.

9.3   In the Quiet of Ariccia

'This morning at 4:30 in his home in Via in Lucina no. 17, the Senator Vito Volterra son of Abram, of the Jewish race, died. Signed: Palma'. This was the report sent by the police station in Rome to the Ministry of the Interior on 11 October 1940 to inform them that the dossier being kept on Vito Volterra could finally be closed.

But surveillance would accompany the opponent of the regime even to his grave. Thus the surveillance reports describe the various phases of the funeral, which even included an informal passage by the church of Santa Maria in Via, probably to receive a brief salutation on the part of Vatican authorities. The hearse then left Rome to go to Ariccia. The people of that town, who had grown fond of the authoritative scientist, braved the ire of Fascist authorities by taking the coffin from the hearse and carrying it on their shoulders in a crowded procession, to the corner of the cemetery where Volterra is still buried today. His daughter-in-law Nella had this to say: 'I can still see myself in October 1940 in a country cemetery where we accompanied my father-in-law, who left us forever at the most tragic moment, when there was not even a glimmer of light that allowed us to think that it might be possible to return to freedom once again. He, in a period in which the majority of citizens had compromised their consciences and changed sides every time the wind changed, was one of the few who had been able to leave to his family and to humanity a luminous example of rectitude, and didn't live to see the fall of the dictatorship!'

Volterra was remembered by the numerous foreign academies of which he had been a member, and was commemorated by the small communities of Italian mathematicians living in various South American countries. The *Bollettino* of the Unione Matematica Italiana broke the silence, and eulogised him, but cautiously. Mathematics historian Gino Loria wrote to Virginia Volterra about this, 'You must

bear in mind the limit imposed on the number of pages in scientific journals. And in the second place the need to take the utmost care to avoid touching on certain themes'. This was a healthy and correct (?) lesson in political realism. In Italy, the Pontificia Accademia delle Scienze, taking advantage of its particular status, was the only one to organise a solemn ceremony, at the opening of the 1941–1942 academic year. The eulogy was entrusted to Carlo Somigliana. The extract of his talk was used by Virginia Volterra to render homage to the memory of her husband and distributed to the numerous Italian and foreign scholars who had remained in contact with Volterra during the years of persecution. Replying with a note of thanks, Benedetto Croce observed, 'It is also significant that this dutiful office to his memory found a place only in the Accademia Pontificia'. It would not be until the inauguration of the reconstituted Accademia dei Lincei in 1946 that Volterra was finally commemorated as he deserved to be, in the seat of an Italian institution, eulogised by Guido Castelnuovo.

And now the word passes to future generations. It is up to them to decide if, and to what extent, Volterra's intelligence and faithfulness to his principles can be useful for the future of Italy.

Appendix I

Significant Dates in Italian History 1849–1945

The first part of this brief chronology is limited to key dates in the history of Italy during the period from 1849 to 1945. The second part briefly outlines the various phases of Italian Fascism.

1849	Founding of the Roman Republic (9 February); occupation of Rome by French troops (3 July); suppression of the revolution (April–August)
1852–1861	Cavour Prime Minister (with interruptions)
1859	War between Franco-Piedmontese forces and the Austrians (the Second Italian War of Independence); Treaty of Zurich (10 November); Lombardy is ceded to the Kingdom of Sardinia
1860	Garibaldi's 'Expedition of the Thousand' through Sicily and Southern Italy
1861	Victor Emmanuel III becomes King of Italy (17 March). The Kingdom of Italy comprised three distinct, historically developed areas: the small northern Italian states; the whole of central Italy, including all but a small portion of the Papal States; and the feudal state that dominated the southern part of the peninsula
1864	The Italian government and Napoleon III sign the 'September Convention' (15 September) regarding the 'Roman Question'. France agrees to withdraw its troops from Rome within 2 years, Italy agrees to refrain from any military attack on Rome and respect the territorial integrity of the Papal States. A secret clause specifies transferring the Italian capital from Turin to a city to be determined. For strategic reasons, Florence is declared the capital shortly after
1866	Italy takes part in the war between Prussia and Austria (Third War of Italian Independence); the Veneto is annexed to the Kingdom of Italy
1870	French troops withdraw from Rome, Italian troops enter, completing the unification of Italy (20 September); Rome is declared the capital (9 October)
1882	The Triple Alliance is formed (Germany, Austria-Hungary, Italy)
1887	Crispi becomes Prime Minister
1890	An Italian colony is established in Eritrea
1892	Founding of the Italian Socialist Party (*Partita Socialista Italiana, PSI*)
1895–1896	Death of Crispi (5 March 1895); First Italo-Ethiopian War
1903–1914	Giolitti is Prime Minister (with interruptions)

(continued)

1911	Tripoli is annexed
1912	Treaty of Lausanne between Italy and the Ottoman Empire: Libya is freed from Ottoman dominance and becomes de facto part of Italy
1914	Outbreak of World War I. Italy declares neutrality (3 August). Struggle between neutralists (Giolitti) and interventionists (D'Annunzio, Mussolini). Salandra succeeds Giolitti as Prime Minister. With Foreign Minister Sonnino, Salandra attempts to enforce Italian territorial claims
1915	Treaty of London (26 April) between Italy and the Triple Entente; Italy enters World War I (23 May)
1918	Victory of the Italian troops at Vittorio Veneto (30 October); Armistice between Italy and Austria signed (3 November)
1919	Catholic priest Luigi Sturzo founds the Italian People's Party (*Partito Popolare Italiano, PPI*); first appearance of Mussolini's 'blackshirts'
1920	Socialist strikes in Milan and Torino. Mistrustful of the government, property owners hired groups of Fascists to protect them against leftist forces
1921	Founding of the National Fascist Party (*Partito Nazionale Fascista, PNF*). During the weak governments of Bonomi (1921–1922) and Facta (Feb. to Oct. 1922), the fascists take 'direct action': threats, use of force, elimination of the provincial bureaucracy in Northern Italy. Industrialists and militarists sympathize with Fascism
1922	'March on Rome' (27–29 October). Victor Emmanuel III invites Mussolini to form a new government, composed of ideologically allied Fascists and members of other parties. Mussolini assumes power (30 October)
1923	'Gentile Reform'. Neo-idealist philosopher Giovanni Gentile carries out this reform as Minister for Public Education of Mussolini's first cabinet
1924	Kidnapping and murder of anti-Fascist politician Matteotti, leader of the United Socialist Party (10 June); outbreak of the Matteotti crisis; formation of the Aventine Seccession (27 June) against Fascism
1925–1926	'Christmas Eve Law' (1925) names Mussolini Head of the Government. Mussolini abolishes any restraint, constitutional or conventional, on his power; opposition parties are outlawed
1928	Parliamentary elections are abolished. Members of Parliament were to be chosen by the 'Grand Council of Fascism' (*Gran Consiglio del Fascismo*) from a list of candidates proposed by the various bodies of Parliament and approved by referendum
1929	Lateran Treaty between Italy and the Vatican (11 February)
1935–1936	Second Italo-Ethiopian War: military occupation of Ethiopia and its annexation to Italian East Africa; League of Nations imposes economic sanctions on Italy
1936	Mussolini announces the 'Rome-Berlin Axis'
1936–1939	German-Italian military intervention in support of Franco in Spain
1938	The promulgation of racial laws in Italy (6 October). Explicitly racist and anti-Jewish measures are articulated in a document outlining 'provisions for the protection of the Italian race'
1939	Italian invasion of Albania; German-Italian military agreement ('Pact of Steel', 22 May)
1940	Italy enters into World War II on the side of Germany (11 June); invasion of British Somaliland, the beginning of the campaign in North Africa (from Feb. 1941 together with the German Africa Corps); invasion of Greece (28 October)

1941	Surrender of Italian forces in Somaliland, Eritrea and Ethiopia (January-October); Italy declares war on the Soviet Union (22 June)
1942	Military defeat of German-Italian forces in El-Alamein (4 November)
1943	Surrender of the last German and Italian units fighting on African soil (13 May); the Allies land in Sicily (10 July); fall of Mussolini (25 July); Italy's surrender is announced (8 September); German forces occupy northern and central Italy; Mussolini proclaims the Italian Socialist Republic (*Repubblica Sociale Italiana*), called the Salò Republic (15 September)
1945	Popular uprising in northern Italy; Milan, Genoa and Torino are liberated (late April); Mussolini is executed (27 April); German forces in northern Italy surrender; the end of the Italian Social Republic

The rise, rule and demise of Fascism must be understood as a historical process in which several phases can be clearly distinguished. The following five phases can identified:

- The period pre-1919: the phase leading to the rise of Fascism;
- The period 1919–1922: the phase when Fascism is a 'movement'. This phase ends with the 'March on Rome' and the coming of Fascism to power.
- The period 1922–1929: the crucial transition phase in which Fascism is consolidated and develops into a dictatorship.
- The period 1929–1943: the actual phase of the Fascist dictatorship.
- The period 1943–1945: the fall of Fascism.

Appendix II

Significant Dates in the Life of Vito Volterra 1860–1940

1860	Volterra is born on 3 May in Ancona, the son of Abramo Volterra und his wife Angelica Almagià
1862	Death of his father
1878	Volterra enrols in the Faculty of Sciences at the University of Pisa
1879	Admission to the elite *Scuola Normale Superiore* in Pisa
1881	Volterra discovers a nowhere dense set of strictly positive outer measures
1882	Volterra constructs a differentiable function whose (bounded) derivative is not integrable
1882	Volterra writes his dissertation on hydrodynamics under the advisement of Enrico Betti; he becomes Betti's assistant
1883	Named Professor of rational mechanics in Pisa
1887	Volterra introduces the first generalized functions that will be directly related to functional analysis, which he called 'functions of lines'; awarded the Gold Medal of the *Società dei XL*
1888	Named corresponding member of the *Accademia dei Lincei*; Named a member of the *Société mathématique de France*; Volterra travels to Paris and meets Poincaré
1891	Named a member of the *Circolo matematico di Palermo*; 1-month study period in Germany; at the Universities of Göttingen and Berlin Volterra meets, among others, H. A. Schwarz and L. Kronecker
1892	Following Betti's death Volterra is elected Dean of the Faculty of Sciences at the University of Pisa; his paper 'Sur les vibrations lumineuses dans les milieux biréfringents' is published in *Acta Mathematica*
1893	Volterra leaves Pisa and moves to Torino, where he accepts a position as professor of higher mechanics and rational mechanics at the University of Torino
1894	Named a member of the *Società dei XL*
1895	Awarded the National Prize for Mathematics of the *Accademia dei Lincei*; beginning of the dispute with Giuseppe Peano, which lasts for some months
1896	First publications on integral equations
1897	Co-founds the Italian Physical Society; named a member of the *Astronomischen Gesellschaft* in Heidelberg; attends the International Congress of Mathematicians in Zurich

(continued)

1898	French mathematician René Baire spends several months in Torino with Volterra; Volterra named a member of the *Accademia dei Lincei*
1900	Becomes a professor of mathematical physics at the University of Rome; marries Virginia Almagià; gives keynote lecture entitled 'Betti, Brioschi, Casorati, trois analystes italiens et trois manières d'envisager les questions d'analyse' at the International Congress of Mathematicians in Paris
1901	Gives the inaugural lecture of the academic year at the University of Rome, discussing 'attempts at applying mathematics to biological and sociological sciences' ('Sui tentativi di applicazione delle matematiche alle scienze biologiche e sociali'); travels to England (London, Oxford, Cambridge)
1902	Travels to Germany, Denmark, Sweden and Norway
1904	Awarded an honorary doctorate from the University of Cambridge; attends the International Congress of Mathematicians in Heidelberg; elected corresponding member of the French *Académie des Sciences*; Volterra is awarded the government contract for the reorganization of the Politecnico di Torino
1905	At the invitation of Swedish King Oscar II Volterra gives a series of lectures in Stockholm on partial differential equations; appointment as Senator of the Kingdom of Italy; publication of the first work on the theory of elasticity
1906	Volterra proposes founding an Italian Association for the Advancement of Science (*Società Italiana per il Progresso delle Scienze, SIPS*); at the inaugural congress of the SIPS he is elected president
1907	Elected Dean of the Faculty of Sciences at the University of Rome
1908	Volterra organises the International Congress of Mathematicians in Rome, and gives the keynote lecture; elected a member of the Imperial Academy of Sciences in St. Petersburg and the Royal Academy of Sciences in Sweden
1909	First publications on 'hereditary mechanics' (mechanics of materials that retain a 'memory' of previous effects); Volterra founds the Thalassographic Committee within the SIPS to study ocean currents and fishing; lecture series in Rome on integral equations and integro-differential equations; first voyage to the United States and Canada; lectures series on mathematical physics and award of an honorary degree, Clark University in Massachusetts
1910	Fellow of the Royal Society; travels to South America
1911	Elected a member of the American National Academy of Sciences
1912	Lectures at the Sorbonne in Paris on functions of lines; second voyage to the United States; lectures at University of Illinois, Columbia University, Princeton, Harvard and Rice Institute
1915	Following the outbreak of World War I, Volterra volunteers for the Italian army
1916	Death of his mother
1917	Appointed head of the Office of Inventions, founded to coordinate the military, industry and universities
1918	Volterra converts the Office of Inventions into the Office of Inventions and Research; attends the meetings in London and Brussels of the Inter-Allied Research Council
1920	Keynote lecture at the International Congress of Mathematicians in Strasbourg
1923	Volterra elected the first president of the National Council for Research (*Consiglio Nazionale delle Ricerche, CNR*); also elected president of the *Accademia dei Lincei*
1925	First publications on population dynamics
1926	With the rise of the Fascist regime, Volterra steps down at the end of the first 3-year period from his presidency in the CNR and in the *Accademia dei Lincei*

1928	Keynote lecture at the International Congress of Mathematicians in Bologna
1931	Publication of the *Leçons sur la théorie mathématique de la lutte pour la vie*; Volterra refuses to swear allegiance to the Fascist regime
1934	Volterra is removed from the *Accademia dei Lincei*
1936	Publication of vol. 1 of *Théorie générale des fonctionnelles*, written in collaboration with J. Pérès; Volterra nominated a member of the *Pontificia Accademia delle Scienze*, reorganised by Pope Pius IX
1938	Volterra is removed from the *Istituto Lombardo di Scienze e Lettere* because 'he belonged to a non-Aryan race'
1940	Volterra dies on 11 October in Rome

References

Albers DJ, Alexanderson GL, Reid C (1987) International mathematical congresses. An illustrated history. Springer, Berlin/Heidelberg/New York

Bartocci C, Odifreddi P (eds) (2007) La Matematica. Vol. I: I luoghi e i tempi. Einaudi, Torino

Battimelli G (1996) Senza alcun vincolo ufficiale: Tullio Levi-Civita e i congressi internazionali di meccanica applicata. Rivista di Storia della Scienza 110(4):51–80

Berengo M (2004) Cultura e istituzioni nell'Ottocento italiano. Il Mulino, Bologna

Bianchi L (1953) In memoria di Ulisse Dini. Cited in: Dini U, Opere. vol 1. Cremonese, Rome

Boatti G (2001) Preferirei di no. Le storie dei dodici professori che si opposero a Mussolini. Einaudi, Turin

Bottazzini U (1994) The Italian states. In: Grattan-Guinness 1994:1495–1504

Bottazzini U (2002) Italy. In: Dauben-Scriba 21:61–95

Boyer CB (1949) The history of calculus. Dover Publications, New York

Boyer CB (2007) Storia del calcolo. Bruno Mondadori, Milan (Italian ed. of [Boyer 1949], translated and edited by A. Guerraggio)

Boyer CB, Merzbach UC (1991) A history of mathematics. Wiley, New York

Brown M (1983) Why the percentage of sharks caught in Mediterranean Sea rose dramatically during World War I. In: Brown M, Coleman CS, Grew DA (eds) Differential equation models. Springer, Berlin

Cardillo M (1987) Tra le quinte del cinematografo: cinema, cultura e societa' in Italia 1900–1937. Edizioni Dedalo S.p.A, Bari

Conti E (1986) Dal taccuino di un borghese. Il Mulino, Bologna

Cross JJ (1994) Theories of elasticity. In: Grattan-Guinness 1994:1023–1033

Dauben JW, Hunger Parshall K (2007) L'evoluzione della ricerca universitaria: Johns Hopkins, Chicago and Berkeley. In: Bartocci C, Odifreddi P (eds) La matematica, vol 1. Einaudi, Turin

Dauben JW, Scriba CJ (eds) (2002) Writing the history of mathematics: its historical development. Birkhäuser Verlag, Basel/Boston/Berlin

Di Sieno S, Guerraggio A, Nastasi P (eds) (1998) La matematica italiana dopo l'Unita. Gli anni tra le due guerre mondiali. Marcos y Marcos, Milan

Fichera G (1994) Vito Volterra and the birth of functional analysis. In: Pier 1994:171–183

Fre'chet M (1906) Sur quelques points du calcul fonctionnel. Rendiconti Circolo matematico di Palermo 22:1–74

Furedi F (2007) Che fine hanno fatto gli intellettuali? Cortina, Milan (English trans. of: Where have all the intellectuals gone? Confronting twenty-first century philistinism. Continuum International Publishing Group 2004)

Galilei G (1623) Il Saggiatore. Rome

Galuzzi M, Guerraggio A (1980) Volterra e l'Economia matematica. Giornale degli economisti e Annali di Economia 1(1–12):803–814

Garding L (1989) History of the mathematics of double refraction. Arch Hist Exact Sci 40 (4):355–385

Goodstein JR (2007) The Volterra Chronicles. The life and times of an extraordinary mathematician 1860–1940. American Mathematical Society, Providence

Grattan-Guinness I (ed) (1994) Companion encyclopedia of the history and philosophy of the mathematical sciences, vol 1. Routledge, London/New York

Gray J (2000) The Hilbert challenge. Oxford University Press, Oxford

Guerraggio A (1984) Le memorie di Volterra e Peano sul movimento dei poli. Arch Hist Exact Sci 31(2):97–126

Guerraggio A (1986) Economia e matematica in Italia tra Ottocento e Novecento. Scientia 121:13–29

Guerraggio A (1990) L'economia matematica in Italia tra le due guerre: Luigi Amoroso. Quaderni di storia dell'economia politica

Guerraggio A, Nastasi P (2005a) Italian mathematics between the two world wars. Birkhäuser Verlag, Basel/Boston/Berlin

Guerraggio A, Nastasi P (2005b) Matematica in camicia nera. Bruno Mondadori, Milan

Guerraggio A, Nastasi P (2008) Roma 1908: il Congresso internazionale dei matematici. Bollati Boringhieri, Torino

Hadamard J (1945) The psychology of the invention in the mathematical field. Princeton University Press, Princeton

Hardy GH (1967) A mathematician's apology. Cambridge University Press, London

Israel G (1982) Le equazioni di Volterra e Lotka; una questione di priorità. In: Atti del Convegno "La storia delle matematiche in Italia", Cagliari, pp 495–502

Israël G (1994) Mathematical biology. In: Grattan-Guinness 1994:1275–1280

Israël G, Millán Gasca A (2002) The biology of numbers. The correspondence of Vito Volterra on mathematical biology. Birkhäuser Verlag, Basel/Boston/Berlin

Kennedy HC (1974) Giuseppe Peano. Birkhäuser Verlag, Basel

Klein F (1979) Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert. Teile 1 und 2 (reprint of the editions of 1926 (Teil 1) und 1927 (Teil 2)). Springer, Berlin

Kline M (1972) Mathematical thought from ancient to modern times. Oxford University Press, Oxford

Lacaita CG, Silvestri A (2000) Francesco Brioschi e il suo tempo (1824–1897), vol 1. Saggi Franco Angeli ed., Milan

Lehto O (1998) Mathematics without borders. A history of the International Mathematical Union. Springer, New York

Levi Mortera N (n.d.) Ritorno alla libertà. (This is a private diary that was published after World War II by Volterra's daughters Laura, and Virginia)

Linguerri S (2005) Vito Volterra e il Comitato talassografico italiano. Olschki, Florenz

Love A (1920) A treatise on the mathematical theory of elasticity. Cambridge University Press, Cambridge

Lützen J (1994) Integral equations. In Grattan-Guinness 1994:385–394

Mazliak L, Tazzioli R (2009) Mathematicians at war. Volterra and his French colleagues in World War I. Springer, Berlin

Nastasi P, Tazzioli R (2005) Toward a scientific and personal biography of Tullio Levi-Civita (1873–1941). Hist Math 32(2):203–236

Paoloni G (1990) Vito Volterra e il suo tempo (1860–1940). Mostra storico-documentaria. In: Paoloni G (ed) Exhibition catalogue. Accademia Nazionale dei Lincei, Rome

Palladino F, Palladino N (eds) (2006) Dalla "moderna geometria" alla "nuova geometria italiana" Viaggiando per Napoli, Torino e dintorno. Olschki, Florenz

Paoloni G, Simili R (eds) (2004) I Lincei nell'Italia unita. Giorgio Bretschneider, Rome

Pastrone F (1998) Fisica matematica e meccanica razionale. In: Di Sieno S, Guerraggio A, Nastasi P (eds), pp 581–504

Pastrone F, Tonon ML (1982) Le origini della teoria delle dislocazioni: i contributi italiani. Bollettino dell'Unione Matematica Italiana

Peano G (1895) Il principio delle aree e la storia di un gatto. Rivista di matematica 5:31–32

Pier J-P (ed) (1994) Development of mathematics. Birkhäuser Verlag, Basel/Boston/Berlin

Puppi G (1992) Vito Volterra e la fisica del suo tempo. In: Atti del Convegno internazionale in memoria di Vito Volterra. Acc. Lincei, Rome

Ragionieri E (1976) La storia politica e sociale. In: Storia d'Italia, vol 4, t. III. Einaudi, Torino

Reid C (1996) Hilbert. Springer, Berlin

Reinhardt V (2006) Geschichte Italiens, 3rd edn. Verlag C. H. Beck, München

Scudo FM (1992) Vito Volterra, ecology and the quantification of Darwinism. In: Atti del Convegno internazionale in memoria di Vito Volterra, Rome. Accademia Nazionale dei Lincei, Rome, 8–11 Oct 1990

Segal SL (2003) Mathematicians under the Nazis. Princeton University Press, Princeton

Segrè E (1995) Autobiografia di un fisico. Il Mulino, Bologna

Siegmund-Schultze R (1994) Functional analysis. In: Grattan-Guinness 263 1994:375–384

Siegmund-Schultze R (2004) A non-conformist longing for unity in the fractures of modernity: towards a scientific biography of Richard von Mises (1883–1953). Sci Context 17(3):333–370

Siegmund-Schultze R (2009) Mathematicians fleeing from Nazi Germany. Princeton University Press, Princeton/Oxford

Simili R, Paoloni G (eds) (2001) Per una storia del Consiglio nazionale delle ricerche. GLF editori Laterza, Rom/Bari

Smith DE (1910) The International Commission on the teaching of mathematics. Am Math Mon 17(1):1–8

Tomassini L (2001) Le origini. In: Simili und Paoloni 2001: 5–71

Turi G (2006) Giovanni Gentile. Una biografia, 2nd edn. Utet, Torino

Vailati G (1987) Scritti di Giovanni Vailati. In; Quaranta M (ed) Forni, Bologna

van Dalen D, Monna AF (1972) Sets and integration. An outline of the development. Wolters-Noordhoff Publishing, Groningen

Volterra V (1881a) Sul potenziale di un'ellissoide eterogenea sopra sè stessa. Nuovo Cimento 9:221–229

Volterra V (1881b) Alcune osservazioni sulle funzioni punteggiate discontinue. G Mat 19:76–87

Volterra V (1882a) Sopra alcuni problemi di idrodinamica. Nuovo Cimento 12:65–96

Volterra V (1882b) Sui principii del calcolo integrale. G Mat 19:333–372

Volterra V (1883a) Sopra alcune condizioni caratteristiche delle funzioni di una variabile complessa. Ann Mat Pura Appl 11(2):1–55

Volterra V (1883b) Sulle figure elettrochimiche du A. Guebhard. Atti Accad Sci Torino 18:329–336

Volterra V (1884) Sopra un problema di elettrostatica. Nuovo Cimento 16:49–57

Volterra V (1887a) Sopra le funzioni che dipendono da altre funzioni. R C Accad Lincei 3(4):97–105, 141–146 and 153–158

Volterra V (1887b) Sopra le funzioni dipendenti da linee. R C Accad Lincei 3(4):225–230, and 274–281

Volterra V (1887c) Sopra un'estensione della teoria di Riemann sulle funzioni di variabili complesse. I. RC Accad Lincei 3(4):281–287

Volterra V (1888) Sopra un'estensione della teoria di Riemann sulle funzioni di variabili complesse. II, III. R C Accad Lincei 4(4):107–115, and 196–202

Volterra V (1889a) Sur une généralisation de la théorie des fonctions d'une variable imaginaire. 1er Mémoire. Acta Math 12:233–286

Volterra V (1889b) Delle variabili complesse negli iperspazi. R C Accad Lincei 5(4):158–165, and 291–299

Volterra V (1890) Sopra un'estensione della teoria di Jacobi-Hamilton del calcolo delle variazioni. RC Accad Lincei 6(4):127–138

Volterra V (1892) Sur les vibrations lumineuses dans les milieux biréfringents. Acta Math
        16:153–215

Volterra V (1896) Sulla inversione degli integrali definiti. Atti Accad (Torino) 31:311–323,
        400–408, 537–567 and 693–798

Volterra V (1898a) Sul fenomeno delle "seiches". Nuovo Cimento 8:270–272

Volterra V (1898b) Sur la théorie des variations des latitudes. Acta Math 22:201–296

Volterra V (1906a) L'economia matematica ed il nuovo manuale del Prof. Pareto. Giornale degli
        Economisti 32:296–301

Volterra V (1906b) Leçons sur l'intégration des équations differentielles aux dérivées partielles,
        professées à Stockolm (février-mars 1906) sur l'invitation de S. M. le Roi de Suède. Uppsala

Volterra V (1907) Sur l'équilibre des corps e´lastiques multiplement connexes. Annales
        scientifiques de l'Ecole Normale Supérieure 24(sIII):401–517

Volterra V (1909) Sulle equazioni integro-differenziali della teoria dell'elasticità. Rendiconti
        Accademia dei Lincei 18(2):295–301

Volterra V (1910) Sopra le funzioni permutabili. Rendiconti Accademia dei Lincei 19:425–437

Volterra V (1912a) L'application du calcul aux phénomènes d'hérédité. Revue du mois
        13:257–274

Volterra V (1913a). In: Pérès J (ed) Leçons sur les fonctions de lignes, professées à la Sorbonne en
        1912. Collection Borel, Gauthier-Villars, Paris

Volterra V (1913b) In; Tomassetti M, Zarlatti FS (eds) Lec¸ons sur les équations integrales et les
        équations intégro-differentielles, professées à la Faculté des Sciences de Rome en 1910.
        Collection Borel, Gauthier-Villars, Paris

Volterra V (1912) Trois leçons sur quelques progrès récents de la physique mathématique.
        Lectures delivered Clark University, Department of Physics, September 1909. Published by
        Clark University

Volterra V (1916a) Metodi di calcolo degli elementi di tiro per artiglieria aeronautica. Rendiconti
        dell'Istituto centrale aeronautico Roma V, 16 (50 pages und 5 fold-out plates)

Volterra V (1916b) Teoria delle potenze dei logaritmi e delle funzioni di composizione. Mem
        Accad Lincei 11(5):167–269

Volterra V (1920a) Saggi scientifici. Bologna. Newly edited by R. Simili, Zanichelli, Bologna,
        1990

Volterra V (1920b) Osservazioni sul metodo di determinare la velocità dei dirigibili. Rassegna
        marittima e aeronautica illustrata, Rome

Volterra V (1921) Funzioni di linee, equazioni integrali e integro-differenziabilii. Anales de la
        Sociadad Cientifica Argentina

Volterra V (1924) (with J. Pérès) Leçons sur la composition et les fonctions permutables. Paris

Volterra V (1926) Fluctuations in the abundance of a species considered mathematically. Nature
        118:558–560

Volterra V (1927a) Teoria de los funcionales y de las ecuaciones integrales e integro-
        differenciales. Conferencias explicadas en la Facultad de la Ciencias de la Universidad,
        1925, redactadas por L. Fantappié, Madrid

Volterra V (1927b) Variazioni e fluttuazioni del numero d'individui in specie animali conviventi.
        R. Comit. Talass. Italiano, Memoria CXXXI, Venice

Volterra V (1927c) Una teoria matematica sulla lotta per l'esistenza. Scientia 41:85–102 (French
        trans: Une théorie mathématique de la lutte pour la vie. Scientia 41:33–48)

Volterra V (1927d) Sulle fluttuazioni biologiche. Rendiconti Accademia dei Lincei 5(s.6):3–10

Volterra V (1927e) Leggi delle fluttuazioni biologiche. Rendiconti Accademia dei Lincei 5
        (s.6):61–67

Volterra V (1927f) Sulla periodicità delle fluttuazioni biologiche. Rendiconti Accademia dei
        Lincei 5(s.6):465–470

Volterra V (1931) Leçons sur la théorie mathématique de la lutte pour la vie. Gauthier-Villars,
        Paris

Volterra V (1932) Le calcul des variations, son évolution et ses progrès, son rôle dans la physique mathématique. Lectures given in 1931 at the Fac. of Sci., Univ. Charles, Prague and at the Fac. of Sci., Univ. Masaryk, Brno. Prague and Brno. Czech trans. Căsopis 62 (1933), 93–116 and 201–227

Volterra V (1935) (with U. D'Ancona), Les associations biologiques étudiées au point de vue mathématique. Hermann, Paris

Volterra V (1936) (with J. Pérès): Théorie générale des fonctionnelles. Tome I: Généralités sur les fonctionelles. Théorie des équations intégrales. Collection Borel, Paris

Volterra V (1937) Applications des Mathématiques à la Biologie. L'Enseignement mathématique 36:297–330

Volterra V (1938a) Rotation des corps dans lesquels existent des mouvements internes. In: Costabel P (ed) Gauthier-Villars, Paris

Volterra V (1938b) (with B. Hostinsky): Opérations infinitésimales linéaires. Applications aux équations différentielles et fonctionnelles. Collection Borel, Paris

Volterra V (1954/1962) Opere matematiche. Memorie e note, 5 vols, Rome

Volterra V, Volterra E (1960) Sur les distorsions des corps élastiques (théorie et applications). Gauthier-Villars, Paris

Weingarten J (1901) Sulle superfici di discontinuità nella teoria dell'elasticità dei corpi solidi. Rend Accad Lincei 10(5):57–60

Wright H (1994) Explorer of the universe. A biography of George Ellery Hale. American Institute of Physics, New York

Yandell BH (2002) The honors class. Hilbert's problems and their solvers. A. K. Peters, Natick

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